Math 163 Midterm 3 Review Sheet.

To study: Some sample problems are provided below, but they will probably not provide sufficient review/practice and might not be completely representative of the exam. You should also look at unassigned book problems from the various sections. Good luck studying!

1. Describe a non-zero vector function $\mathbf{r}(t)$ in $\mathbb{R}^3$ whose acceleration vector $\mathbf{a}(t)$ satisfies $\mathbf{a}(t) = -\mathbf{r}(t)$.
2. Consider the graph of $\mathbf{r}(t) = \langle t, t^2, 0\rangle$. What are the vectors $\mathbf{T}$ and $\mathbf{N}$ at the point where $t = 0$?
3. Consider the function $\mathbf{r}(t) = \langle 3e^{t/2}, 4e^{-t/2}\rangle$.
   (a) Compute $\mathbf{v}(t)$ and $\mathbf{a}(t)$.
   (b) Are $\mathbf{r}(t)$ and $\mathbf{a}(t)$ parallel, perpendicular, the same, or none of the above?
   (c) Is the unit tangent vector at $t = 0$ orthogonal to the acceleration vector at $t = 0$?
4. A malfunctioning robotic mountain goat is following a track up a mountain; his coordinates (in hectometers, as a function of time in hours) are given by $\mathbf{r}(t) = \langle e^t, e^{-t}, \sqrt{2} \cdot t\rangle$.
   (a) What is the change in his altitude from $t = 0$ to $t = 10$?
   (b) What is his speed in the x-direction when $t = 4$?
   (c) Determine the total distance traveled by the robotic goat from $t = 0$ to $t = 4$.
5. Let $\mathbf{r}(t) = \langle \sin 2t, 3t, \cos 2t\rangle$ for $-\pi \leq t \leq \pi$ be the position vector of a particle at time t.
   (a) Show that the velocity and acceleration vectors are always orthogonal.
   (b) Is there any time t for which $\mathbf{r}$ and $\mathbf{v}$ are orthogonal? If so, find *all* such values of t.
6. The position of a particle at time t is given by $\mathbf{r}(t) = \langle t^3 - 3t, t^2\rangle$. For each value of t for which the particle crosses the y-axis, find the unit tangent vector.
7. Evaluate $\int_0^1 \left(t\mathbf{i} - 2t^2\mathbf{j}\right)\, dt$.
8. Find $\mathbf{r}(1)$ if $\mathbf{r}'(t) = t^2\mathbf{i} + t^3\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{i}$.
9. Let $\mathbf{u}(t) = 2t\mathbf{i} + \sin t\mathbf{j} - \cos t\mathbf{k}$ and $\mathbf{v}(t) = \mathbf{i} + t^2\mathbf{j} - t\mathbf{k}$. Find $\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right]$.
10. Let $\mathbf{u}(t) = \left\langle -\sqrt{t}\sin t, t, t^{2/3}\right\rangle, \mathbf{v}(t) = \left\langle -\sqrt{t}\sin t, \cos^2 t, -t^{1/3}\right\rangle$. Find $\frac{d}{dt}\left[\mathbf{u}(t) \cdot \mathbf{v}(t)\right]$ and $\frac{d}{dt}\left[\mathbf{u}(t) \cdot \mathbf{u}(t)\right]$.
11. Find the length of the curve $\mathbf{x}(t) = \langle 5t, \sin t, \cos t\rangle$ from $t = 0$ to $t = 2$.
12. Show that if $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are parallel at some point on the curve described by $\mathbf{r}(t)$, then the curvature at that point is 0. Then give an example of a curve $\mathbf{r}(t)$ for which $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are *always* parallel.
13. A cannon sits on top of a vertical tower 264 feet tall. It fires a cannonball at 80 feet/second. The barrel of the cannon is elevated 30 degrees from the horizontal. Assume the ground around the tower is level.
   (a) How far from the base of the tower will the cannonball land?
   (b) What will be the speed of the cannonball when it lands?

(c) What will be the maximum height achieved by the cannonball (relative to the ground)?

14. Sketch the domain of the function $f(x,y) = \sqrt{x-y^2}$, then find the range of the function.

15. Describe the level curves of the function $f(x,y) = x^2 + y^2 + 3x - 4y + 73$.

16. Suppose the point (2,3) is on a level curve of the function $f(x,y) = x^2 + 2y$. Explain why the point (4, -3) is on the same level curve.

17. Let $f(x,y) = (x+y)^2$. Find $f_{xx}(3,1)$.

18. Let $f(x,y) = x^3y^2$. Find $f_{yxy}(1,7)$.

19. Let $f(x,y) = \sin(2x+y)$. Find $f_{xy}\left(\pi, \frac{\pi}{2}\right)$.

20. Let $f(x,y) = \left(x^3 + y^4\right)^5$. Find the value of $f_{xy} - f_{yx}$ at (1,2).

21. Let $f(x,y,z) = z^{xy}$ for $z > 0$. Find the functions f_x, f_y, f_z and calculate $f_y(2,1,e)$.

22. Find $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ for $f(x,y) = \dfrac{2x-3y}{3x-2y}$.

23. Let $f(x,y,z) = x\ln\left(yz^2\right)$. Find f_{xy}, f_{xz}, f_{yz}.

24. If g is a differentiable function of one variable and $f(x,y) = g\left(x^2+y^2\right)$, show that $yf_x - xf_y = 0$.

25. Consider the function $f(x,y) = \dfrac{1}{x^2+y^2+1}$.

    (a) Sketch the level curves $f(x,y) = \frac{1}{5}$ and $f(x,y) = \frac{1}{10}$.

    (b) Find a value of k for which the level curve $f(x,y) = k$ consists of a single point.

26. Does the function $f(x,y) = \sin^2\left(xy^2\right)$ satisfy the equation $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = (2x+y)(2y)\cos\left(xy^2\right)\sqrt{f(x,y)}$ when $\sin\left(xy^2\right) \geq 0$?

27. Is it possible to find a function for which it is true that, for all $x > 0$ and $y > 0$, $f_x > 0$ and $f_y < 0$, and $f(x,y) > 0$? If so, give an example. If not, why not?