Math 163 Midterm 2 Review Sheet.

To study: Some sample problems are provided below, but they will probably not provide sufficient review/practice and might not be completely representative of the exam. You should also look at unassigned book problems from the various sections, and definitely you should examine the chapter 12 review starting on page 812. Not every problem in the book's chapter review will be appropriate preparation for the upcoming exam – for example, you will not be tested on torque or ellipsoids.

Sample Problems.

1. Find an equation for the set of points that are equidistant from the point (0,0,0) and (2,4,8). (Hint: the set of points is a plane.)
2. True or false? (Explain!)
   (a) $|\mathbf{v}+\mathbf{w}| = |\mathbf{v}| + |\mathbf{w}|$
   (b) $\dfrac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{u}\cdot\mathbf{w}} = \dfrac{\mathbf{v}}{\mathbf{u}}$
   (c) If the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$ equals the scalar projection of $\mathbf{w}$ onto $\mathbf{v}$, then the lengths of $\mathbf{v}$ and $\mathbf{w}$ are the same.
   (d) Suppose that $\mathbf{v}\cdot\mathbf{w} = \mathbf{w}\cdot\mathbf{u} = \mathbf{u}\cdot\mathbf{v} = 0$. Then $\mathbf{v}\times\mathbf{u}$ is parallel to $\mathbf{w}$.
3. Find the vector projection of $\mathbf{b} = \langle -2, 8, 3\rangle$ onto $\mathbf{a} = \langle 1, -4, 2\rangle$.
4. Describe the graph of $x^2 + y^2 + z^2 - 12x - 6y + 4y = 0$.
5. Find all vectors of length 2 orthogonal to both $\mathbf{i} + 2\mathbf{j}$ and $\mathbf{j} - 3\mathbf{k}$.
6. Find the vector equation for the line through (-2, 4, 10) and (4, 0, -4).
7. Find an equation for the plane through (1,2,3) perpendicular to the line from the previous problem.
8. Find the angle between $\mathbf{v} = 4\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$ (rounded to the nearest degree)
9. Find the direction angles of $\mathbf{v} = \langle 3, -2, 5\rangle$.
10. Suppose $\mathbf{r}(t) = \left\langle \dfrac{1+t}{1-t}, \dfrac{\sin^2 t}{t^2}, e^{-2t} \right\rangle$. Find $\lim\limits_{t\to\infty} \mathbf{r}(t)$ (if it exists).
11. Suppose $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
    (a) Find the area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$.
    (b) Find the volume of the parallelopiped determined by $\mathbf{u}, \mathbf{v}, \mathbf{w}$.
12. The points A(0, 2, 3), B(1, -2, 3), and C(2, 3, -5) form a triangle.
    (a) Find the area of the triangle.
    (b) Find an equation for the plane containing the triangle.
    (c) Find parametric equations for the line where the plane containing the triangle intersects the plane $2x - 2y - z - 3 = 0$

13. Find the point in which the line through the origin perpendicular to the plane $2x - y - z = 4$ meets the plane $3x - 5y + 2z = 6$.

14. Find the distance from the point (6, 0, -6) to the plane $x - y = 4$. (Hint: the shortest line segment connecting the point to the plane will be perpendicular to the plane.)

15. A point moves on a circle of radius 3 centered at (-3, -4, 8) in the plane $z = 8$. It makes one complete revolution every 4 seconds; at $t = 0$ it is at the point (0, -4, 8) and at $t = 1$ it is at (-3, -1, 8). Find a vector function defined for all real numbers that describes the point's trajectory.