Math 163 Midterm 1 Review Sheet. Sample problem solutions.

Warnings: in general, you will need to show more work than I do here! For example, if you say that a series converges because of the Ratio test, you need to SHOW me.

1. It converges to 0 by an application of L'hospital's rule.
2. (a) The series diverges. One way to show this: a basic comparison test with $\frac{1}{n}$.
   (b) It is a telescoping sum which converges to 3/2.
   (c) This converges by the Root Test. But we have not learned how to find the value of this sum.
   (d) This is an alternating series; it converges by the Alternating Series Test. To what, I do not know.
   (e) This converges. Try a comparison test with $\dfrac{1}{n^{3/2}}$. I don't know what this converges to.
   (f) This converges; try a limit comparison test with $\dfrac{10^n}{11^n}$. I don't know what this converges to.
3. It converges to e^3.
4. One possibility is to let $a_n = \dfrac{1}{n}$ and $b_n = \dfrac{-1}{n}$.
5. It's a geometric series with $r = -1/4$; it converges to $\dfrac{1}{1-(-1/4)} = 4/5$.
6. Use the Root Test; $\lim\limits_{n\to\infty} \sqrt[n]{3^n|a_n|} = 3 \cdot \dfrac{1}{2} > 1$, so the series diverges.
7. By the Ratio Test, the series converges absolutely for any value of x such that $|x| < 1$. For $x = 1$ this series is just a p-series with p=1/2, which diverges. For $x = -1$ it is an alternating series which converges by the alternating series test, but note that it converges conditionally.
8. $0.99999999\ldots = \dfrac{9}{10} + \dfrac{9}{100} + \dfrac{9}{1000} = 9 \cdot \dfrac{10}{1-\frac{1}{10}} = 1$
9. Use the Ratio test. $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \dfrac{|x|^{2n+3}(2n+1)!}{|x|^{2n+1}(2n+3)!} = \lim\limits_{n\to\infty} \dfrac{x^2}{(2n+3)(2n+2)} = 0$ for all x – hence the series converges for all x.
10. $C + \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)\cdot(2n+1)!} = C + \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} - \frac{x^{15}}{15\cdot 7!}.$