Math 163 Midterm 1 Review Sheet.

For the upcoming midterm, you will need to know:

1. Sequences:
   (a) What they are
   (b) What it means for a sequence to converge
   (c) The limit laws for sequences and how/when to use them
   (d) The Squeeze Theorem
   (e) How to determine if a sequence converges, and if it converges, to what value it converges
   (f) How to work with recursively defined sequences
2. Series:
   (a) What they are
   (b) What is a partial sum
   (c) How to determine if a series converges, and to explain your reasoning
   (d) The harmonic series; geometric series; alternating series; telescoping series; p-series
   (e) The rules for manipulating series in the box on page 693
   (f) How to calculate the value of a geometric series (with $|r| < 1$) or a telescoping series
   (g) The various tests for convergence/divergence and how to use them. I may ask you to use a specific test on a particular series, even if that test is not the one you yourself would choose to use.
   (h) Absolute convergence
3. Power Series:
   (a) What they are
   (b) How to find the radius and interval of convergence
   (c) Term-by-term integration and differentiation
   (d) Using substitution with power series – e.g., to show that $\frac{1}{1-x^2} = \sum_{n=0}^{\infty} (x^2)^n$
   (e) Although I will not test you on the properties of Taylor/Maclaurin series in general, they provide good and helpful examples of power series

Warning: If a particular term does not appear on the outline above, that does not mean you won't be tested on it!

To study: Some sample problems are provided below, but they will probably not provide sufficient review/practice and might not be completely representative of the exam. You should also look at unassigned book problems from the various sections, and definitely you should examine the chapter 11 review starting on page 758. Not every problem in the book's chapter review will be appropriate preparation for the upcoming exam – for example, you will not be tested on 'error' or remainders or binomial series.

Sample Problems.

1. Determine whether $a_n = \dfrac{\ln(n^2)}{n}$ converges or diverges, and if it converges, find the limit.

2. For each of the following series, determine whether it converges or diverges (and explain how you know). If it converges and the sum can be calculated, do so.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n^2)}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{9}{(3n-1)(3n+2)}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}+10^n}{n^{11}+11^n}$

3. Does the sequence $c_n = \left(1+\frac{3}{n}\right)^n$ converge or diverge? If it converges, to what value does it converge?

4. Give an example of divergent series $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ such that $\displaystyle\sum_{n=1}^{\infty} (a_n + b_n)$ converges.

5. Find the sum $1 - \dfrac{1}{4} + \dfrac{1}{16} - \dfrac{1}{64} + \cdots$

6. Suppose $\{a_n\}$ is a sequence such that $\sqrt[n]{|a_n|} = \frac{1}{2}$. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} 3^n a_n$ converges or diverges.

7. For the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ find the interval of convergence, and determine the values of x for which the series converges absolutely and for which the series converges conditionally.

8. Show, using an infinite series, that $0.9999999\ldots = 1$.

9. Show that the power series $\sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ converges for all real numbers x.

10. Use the series from the previous problem to find a power series for $\int \sin x^2 \, dx$. Write your answer in sigma notation – also write the first five terms in the series (including the constant of integration).