

General instructions: give exact answers to all problems unless otherwise noted. Simplify all answers completely.

Math& 163 Big List of Homework, part four.

O. Recommended book problems: 14.6 / 2, 5-8, 14, 15, 16, 19, 20, 29, 30, 32, 35.

P. Recommended book problems: 14.7 / 1, 3, 4, 5-15 odd, 21, 29, 31, 39, 51.

Q. Recommended book problems: 11.1 / 1, 2, 5, 7, 11-13all, 17-22all, 25, 35, 55, 61, 63. Turn in:

1. Determine whether $\left\{ \frac{(-1)^n n^2 + n}{4n^3 + 1} \right\}$ converges or diverges. Explain.

2. Determine whether $a_n = \frac{2e^n}{5^n}$ converges or diverges. Explain.

3. Evaluate $\lim_{n \rightarrow \infty} \frac{\cos n}{n}$.

4. True or false? If false, give a counterexample.

(a) If $\{a_n\}$ is bounded, then it converges.

(b) If $\{a_n\}$ converges, then it is bounded.

(c) If $\{a_n\}$ is not bounded, then it diverges.

(d) If $\{a_n\}$ diverges, then it is not bounded.

R. Recommended book problems: 11.2 / 9, 13, 15, 17, 19, 20, 22, 25, 28, 31, 36, 41, 44, 47, 58a (I don't think you need the fact given in parentheses), 64, 73. I dare you to try number 76. Turn in:

1. What is the sum of $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots$?

2. Find an n such that $s_n > 25$ for $\sum_{n=1}^{\infty} 2^n$.

3. True or false: if the partial sums s_n for $\sum_{n=1}^{\infty} a_n$ are increasing, then $\{a_n\}$ is an increasing sequence. Explain.

4. For each of the following statements, find a counterexample showing that it is false.

(a) If $a_n \rightarrow 0$ then $\sum_{n=1}^{\infty} a_n = 0$.

(b) The 100th partial sum of the infinite series defined by $\{a_n\}$ is a_{100} .

(c) If $a_n \rightarrow 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

5. Find the sum $\sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n}$ or state that it diverges.

6. Express $1.22525252525 \dots = 1.2\overline{25}$ as a ratio of integers.

7. Find the exact value of $1 + \pi^{-1} + \pi^{-2} + \pi^{-3} + \dots$ or state that the sum diverges.

S. Recommended book problems: 11.3 / 1, 2, 4, 7, 11, 12, 17, 19, 21, 22

Turn in: for each series below, determine whether it converges or diverges.

1. $\sum_{n=5}^{\infty} \frac{2}{\sqrt{n-4}}$
2. $\sum_{n=1}^{\infty} \frac{1}{2^{\ln n}}$ (Hint: $2^A = e^{\ln 2^A}$ for any real number A.)
3. $1 + \frac{1}{3\sqrt{3}} + \frac{1}{5\sqrt{5}} + \frac{1}{7\sqrt{7}} + \dots$

T. Recommended book problems: 11.4 / 1, 2, 4, 7, 11, 12, 16, 17, 19, 21, 22, 29. Turn in: Determine if each of the following series converges or diverges. In each case explain your reasoning.

1. $\frac{3}{4} + \frac{5}{9} + \frac{7}{16} + \frac{9}{25} + \dots = \sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2} = \sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$
2. $\sum_{n=1}^{\infty} \frac{5}{5n-1}$
3. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln n}$
4. $\sum_{n=1}^{\infty} \frac{1}{1+2+3+4+\dots+n}$. (Hint: you may find example 4 in Appendix E of your textbook helpful.)

U. Recommended book problems: 11.5 / 2, 4, 7, 10, 11, 13, 15.

Turn in: Determine if each of the following series converges or diverges. In each case explain your reasoning.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{e^{1/n}}{n}$
2. $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{n^2-1}}$

V. Recommended book problems: 11.6 / 1, 2, 3, 5, 6, 8, 9, 12, 13, 17, 20, 22, 27, 29, 31.

Turn in: Determine if each of the following series converges or diverges. In each case explain your reasoning.

1. $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$
2. $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$
3. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
4. $\sum_{n=1}^{\infty} a_n$, where $a_1 = 1$ and $a_{n+1} = \frac{1 + \ln n}{n} \cdot a_n$

(We are not explicitly covering section 11.7 in class, but it wouldn't hurt you to browse through it.)

W. Recommended book problems: 11.8 / 3, 4, 6, 7, 9, 13, 15, 17, 23, 25, 30

Turn in: Find the interval of convergence for each of the series below.

$$1. \sum_{n=1}^{\infty} x^n$$

$$2. 1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 + \dots + \left(-\frac{1}{2}\right)^n (x-2)^n + \dots$$

$$3. \sum_{n=1}^{\infty} (2x-4)^n$$

$$4. \sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$$

$$5. \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n$$

$$6. x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \text{ (Do not use any theorems or information from later sections.)}$$

X. Recommended book problems: 11.9 / 4, 5, 6, 7, 8, 13a, 14, 18, 23, 26

Turn in:

1. Find a power series representation for $\frac{2}{1+3x}$ and determine the interval of convergence.

2. We know that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$. Use this to find the sum of the following series:

(a) $\sum_{n=1}^{\infty} nx^{n-1}$ for $|x| < 1$

(b) $\sum_{n=1}^{\infty} nx^n$ for $|x| < 1$

(c) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(hint: for each of these you will need to use the previous result.)

3. Use term-by-term integration to find a power series representation for $\int \tan^{-1} x \, dx$. Give your answer in sigma notation, but also write down explicitly the first four terms of the series.