General instructions: give exact answers to all problems unless otherwise noted. Simplify all answers completely.

Math& 163 Big List of Homework, part two.

G. Recommended textbook problems: 13.1 / 1, 3, 4, 6, 8, 9, 10, 11, 15, 19-24, 35, 41, 42. Turn in:

1. What is the domain of the vector function $\mathbf{r}(t) = e^t\mathbf{i} + \frac{1}{t}\mathbf{j} + (1+t)^{-3}\,\mathbf{k}$?
2. The vector function $\mathbf{r}(t) = \langle \sin t, 0, 4 + \cos t\rangle$ traces a circle. Find the radius, the center, and the plane containing the circle.

H. Recommended textbook problems: 13.2 / 4, 5, 8, 21, 22, 23, 25, 26, 32 (the angle of intersection of 2 curves is the angle between the 2 tangent vectors at the point of intersection), 33, 35, 37, 39. Turn in:

1. Evaluate $\int_0^1 \left[t^3\mathbf{i} + 7\mathbf{j} + (t+1)\,\mathbf{k}\right]\, dt$
2. Evaluate $\lim_{h\to 0} \frac{\mathbf{r}\,(t+h) - \mathbf{r}(t)}{h}$ for $\mathbf{r}(t) = \left\langle t^{-1}, \sin t, 4\right\rangle$ (Hint: does this limit remind you of something you learned about in Math 151?)
3. Find parametric equations for the line tangent to $\mathbf{r}(t) = \ln t\mathbf{i} + t^{-1}\mathbf{j} + 9t\mathbf{k}$ at the point where $t = 1$.
4. Let $\mathbf{r}(t) = \langle \sin 2t \cos t, \sin 2t \sin t, \cos 2t\rangle$.
   (a) Show that $|\mathbf{r}(t)|$ is constant for all t.
   (b) By example 4 in your book, this means that the position vector and the tangent vector must be orthogonal for all t. Show (without using the result from example 4) that indeed, $\mathbf{r}(t)\cdot\mathbf{r}'(t) = 0$ for all t.
5. A fighter plane, which can only shoot bullets straight ahead, travels along the path $\mathbf{r}(t) = \left\langle 5 - t, 21 - t^2, 3 - t^3/27\right\rangle$. Show that there is precisely one time t at which the pilot can hit a target located at the origin.

I. Recommended textbook problems: 13.4 / 3, 4, 9, 11, 14, 15, 16, 17a, 19, 22. Turn in:

1. A gun fires bullets with an initial velocity of 150 meters/second. You are trying to hit a target 400 meters away. Find two angles of elevation that can be used to hit the target.
2. A medieval city has the shape of a square and is protected by walls with length 1000 feet and height 45 feet. You are the commander of an attacking army and the closest you can get to the wall is 250 feet. Your plan is to set fire to the city by catapulting heated rocks over the wall (with an initial speed of 250 feet per second). At what range(s) of angles should you tell your men to set the catapult? Round to the nearest tenth of a degree. Assume the plane containing the path of the rocks is perpendicular to the wall, and assume you are *not* interested in trying to damage the city walls themselves, just the city within. (This is a fun but challenging problem – I encourage you to work with your fellow students!)

J. Recommended textbook problems: 13.3 / 1, 2, 7 (you may use Simpson's rule with $n = 8$ if you want extra 'street credibility'), 19, 25. Turn in:
Consider $\mathbf{r}(t) = t\,\mathbf{i} + \ln t\,\mathbf{j}$.

1. Describe the curve. Specifically, this is the graph of what famous equation in x and y?
2. Find κ, the curvature, as a function of t, for $t > 0$. (I found the formula for curvature involving the cross-product to be most helpful.)
3. Find the value of t for which the curve has largest curvature.

4. What happens to the curvature as $t \to \infty$?

J2. A super-secret extra credit problem worth one (paper) homework assignment. It will be due the same day assignment **J** is due. It will not be accepted late.

Consider the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, with $a > b > 0$.

1. Describe the curve. What familiar shape is it?
2. Is it true for all t that $\mathbf{r}$ is orthogonal to $\mathbf{T}$?
3. Find κ, the curvature, as a function of t, for $t > 0$.
4. Determine the value(s) of t for which the curvature is a maximum, and for which it is a minimum. What is the maximum curvature? What is the minimum curvature?