W.S.#3

Math 207 Review of Multi-variable Calculus

*1. Find the Maclaurin series for the following functions:

e^x, $\cos x$, $\sin x$, $\cosh x$, $\sinh x$

*2a. Prove that $e^{i\theta} = \cos\theta + i\sin\theta$

b. Use part a to prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ and that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

3a. Prove that $\cos x = \frac{1}{2}(e^{ix} + e^{-ix}) = \cosh(ix)$

b. Prove that $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix}) = \frac{\sinh(ix)}{i}$

c. Prove that $\cosh x = \cos(ix)$

d. Prove that $\sinh x = -i\sin(ix)$

e. Use parts c and d to prove that $\frac{d}{dx}(\sinh x) = \cosh x$.

4. Find the arc length of the curve given parametrically by $x = a\cos^3 t$, $y = a\sin^3 t$, where $a \in R$ is a positive constant.

*5. Let $F(x, y)$ be a function of two indepedent variables with continuous first partial derivatives in some rectangle containing the point (x_0, y_0). If $F(x_0, y_0) = 0$ and $F_y(x_0, y_0) \neq 0$, then by the implicit function theorem we may conclude that the relation $F(x, y) = 0$ defines y impicitly as a function of x in some open interval containing x_0. Prove that it then follows that $\frac{dy}{dx} = \frac{-F_x}{F_y}$.