1a. Verify that the function $y = e^x$ is a solution to the differential equation $\frac{dy}{dx} - y = 0$. Can you find any other solutions?

b. Verify that the function $y = 5e^x$ is a solution to the initial boundary problem

$$\frac{dy}{dx} - y = 0 \ , \ y(0) = 5.$$

c. Verify that the function $y = 8e^x + \frac{1}{2}\sin x - \frac{1}{2}\cos x$ is a solution to the differential equation $\frac{dy}{dx} - y = \cos x$. Can you find any other solutions?

2a. Verify that the function $y = x^2$ is a solution to the differential equation $x\frac{dy}{dx} - 2y = 0$. Can you find any other solutions?

b. Verify that the function $y = 3x^2$ is a solution to the initial boundary problem

$$x\frac{dy}{dx} - 2y = 0 \ , \ y(1) = 3$$

c. Verify that each member of the family of functions $y = cx^2 + \frac{1}{3}x^5$, $c \in \mathbf{R}$, is a solution to the differential equation $x\frac{dy}{dx} - 2y = x^5$.

3. Verify that the implicit function defined by $y^2 = e^{2x} + 3$ is a solution to the initial value problem

$$y\frac{dy}{dx} - e^{2x} = 0 \ , \ y(0) = -2.$$

Write the solution y explicitly as a function of x.

4a. Verify that each member of the two parameter family of functions $y = c_1 \sin 2x + c_2 \cos 2x$, $c_1, c_2 \in \mathbf{R}$, is a solution to the differential equation $\frac{d^2y}{dx^2} + 4y = 0$.

b. Verify that each member of the two parameter family of functions $y = c_1 \sinh 2x + c_2 \cosh 2x$ is a solution to the differential equation $\frac{d^2y}{dx^2} - 4y = 0$.

5. Verify that the implicit function defined by $x + y = \arctan y$ is a solution to the differential equation $y^2\frac{dy}{dx} + y^2 = -1$.

6. Verify that the function $y = e^{x^2}\int_0^x e^{-t^2}\,dt$ is a solution to the initial value problem

$$\frac{dy}{dx} - 2xy = 1 \ , \ y(0) = 0.$$

7a. If $y_1(x)$ is a solution to the differential equation $x^3\frac{dy}{dx} + y\cos x = 0$, is it necessarily true that $cy_1(x)$ is also a solution to the same differential equation for any $c \in \mathbf{R}$?
b. If $y_1(x)$ and $y_2(x)$ are solutions to the differential equation $x^3\frac{dy}{dx} + y\cos x = 0$, is it necessarily true that $y_1(x) + y_2(x)$ is also a solution to the same differential equation.

c. Repeat parts a and b for the differential equation $x^3\frac{dy}{dx} + y\cos x = x$.

d. Repeat parts a and b for the differential equation $y^3\frac{dy}{dx} + y\cos x = 0$.