Math 207 Review of Differential Calculus: An Introduction to the Hyperbolic Trig Functions

Define $\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\cosh x = \frac{1}{2}(e^x + e^{-x})$, and $\tanh x = \sinh x / \cosh x$.

1. Sketch graphs of the functions $y = e^x$ and $y = e^{-x}$ on the same coordinate system. Use these graphs to sketch the graphs of the three hyperbolic trig functions defined above. State the domain and range of each function.

2. Based on your graphs, conjecture which of the above functions are odd and which are even, and prove your assertions.

3a. Prove that $\cosh^2 x - \sinh^2 x = 1$.
b. Prove that $\cosh x \geq 1$ for all $x \in \mathbf{R}$.
c. Solve the equation in part a) for $\cosh x$.

4. Sketch the curve given parametrically by $x = \cosh t$, $y = \sinh t$, $t \in \mathbf{R}$.

5. Find the derivatives of the functions $\sinh x$, $\cosh x$, and $\tanh x$.

6a. Prove that the function $f(x) = \sinh x$ is invertible. *Hint:* What does the derivative of this function tell you? What are the domains and ranges of the functions $f(x) = \sinh x$ and $f^{-1}(x) = \sinh^{-1}(x)$?

b. Compute $\frac{d}{dx}(\sinh^{-1} x)$. Use 3c to simplify your answer.

c. Prove that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ and use this result to compute $\frac{d}{dx}(\sinh^{-1} x)$ and check that your answer agrees with part b.

7. Prove that the function $f(x) = \tanh x$ is invertible. *Hint:* What does the derivative of this function tell you? What are the domains and ranges of the functions $f(x) = \tanh x$ and $f^{-1}(x) = \tanh^{-1}(x)$?

b. Compute $\frac{d}{dx}(\tanh^{-1} x)$. Use a variation of 3a to simplify your answer.

c. Prove that $\tanh^{-1} x = \frac{1}{2}\ln(\frac{1+x}{1-x})$ and use this result to compute $\frac{d}{dx}(\tanh^{-1} x)$ and check that your answer agrees with part b.

DO 7