MATH 99
WINTER 2004

SOLUTIONS FOR SAMPLE TEST of MATH 80 CONCEPTS

1.

(a) $\frac{2}{3}+\frac{3}{4} = \frac{4}{4}\cdot\frac{2}{3}+\frac{3}{4}\cdot\frac{3}{3} = \frac{8}{12}+\frac{9}{12} = \frac{8+9}{12} = \boxed{\frac{17}{12} \text{ or } 1\tfrac{5}{12}}$

(b) $\frac{2}{3}-\frac{3}{5} = \frac{5}{5}\cdot\frac{2}{3}-\frac{3}{5}\cdot\frac{3}{3} = \frac{10}{15}-\frac{9}{15} = \frac{10-9}{15} = \boxed{\frac{1}{15}}$

(c) $\frac{2}{3}\cdot\frac{3}{7} = \frac{2\cdot 3}{3\cdot 7} = \frac{2\cdot \cancel{3}^{1}}{{}_{1}\cancel{3}\cdot 7} = \frac{2\cdot 1}{1\cdot 7} = \boxed{\frac{2}{7}}$

(d) $1\tfrac{2}{3}\div 2\tfrac{3}{4} = \frac{3\cdot 1+2}{3}\div\frac{4\cdot 2+3}{4} = \frac{3+2}{3}\div\frac{8+3}{4} = \frac{5}{3}\div\frac{11}{4} = \frac{5}{3}\cdot\frac{4}{11} = \frac{5\cdot 4}{3\cdot 11} = \boxed{\frac{20}{33}}$

(e) $\frac{26}{80} = \frac{2\cdot 13}{2\cdot 40} = \frac{{}^{1}\cancel{2}\cdot 13}{{}_{1}\cancel{2}\cdot 40} = \boxed{\frac{13}{40}}$

(f) $\frac{84}{91} = \frac{7\cdot 12}{7\cdot 13} = \frac{{}^{1}\cancel{7}\cdot 12}{{}_{1}\cancel{7}\cdot 13} = \boxed{\frac{12}{13}}$

2.

(a) $3x-7=12 \Rightarrow 3x=19 \Rightarrow \boxed{x=\frac{19}{3} \text{ or } 6\tfrac{1}{3}}$

(b) $6x+3=2(3x-7) \Rightarrow 6x+3=6x-14 \Rightarrow 3=-14$ (a contradiction) $\boxed{\text{No Solutions}}$

(c) $4(x+1)=2(2x+2) \Rightarrow 4x+4=4x+4 \Rightarrow 4=4$ (an identity) $\boxed{x \text{ can be any Real number}}$

(d) $\frac{2}{7}(x+9)=19 \Rightarrow \frac{7}{2}\cdot\frac{2}{7}(x+9)=19\cdot\frac{7}{2} \Rightarrow 1(x+9)=\frac{19}{1}\cdot\frac{7}{2} \Rightarrow x+9=\frac{19\cdot 7}{1\cdot 2} \Rightarrow x+9=\frac{133}{2}$

$\Rightarrow x=\frac{133}{2}-9 \Rightarrow x=\frac{133}{2}-\frac{9}{1}\cdot\frac{2}{2} \Rightarrow x=\frac{133}{2}-\frac{18}{2} \Rightarrow x=\frac{133-18}{2} \Rightarrow \boxed{x=\frac{115}{2} \text{ or } 57\tfrac{1}{2}}$

3.

(a) $a^2-9b^2 = (a)^2-(3b)^2 = \boxed{(a-3b)(a+3b)}$

(b) $x^2-12x+35 = \boxed{(x-7)(x-5)}$

(c) $2x^2-4xy-10hx+20hy = 2\left(x^2-2xy-5hx+10hy\right) = 2\left(x\underline{(x-2y)}-5h\underline{(x-2y)}\right) = 2\left((x-2y)(x-5h)\right)$

$= \boxed{2(x-2y)(x-5h)}$

(d) $10x^2-30y^2 = \boxed{10\left(x^2-3y^2\right)}$

(e) $2x^2+5x-12 = \boxed{(2x-3)(x+4)}$

4.

(a) $x^2-12x+35=0 \Rightarrow (x-7)(x-5)=0 \Rightarrow$ Either $x-7=0 \Rightarrow \boxed{x=7}$

or $x-5=0 \Rightarrow \boxed{x=5}$

(b) $x(x+2)=5 \Rightarrow x^2+2x=5 \Rightarrow x^2+2x-5=0 \Rightarrow x=\dfrac{-(2)\pm\sqrt{(2)^2-4\cdot(1)\cdot(-5)}}{2\cdot(1)}$

$\Rightarrow x=\dfrac{-2\pm\sqrt{4+20}}{2} \Rightarrow x=\dfrac{-2\pm\sqrt{24}}{2} \Rightarrow x=\dfrac{-2\pm\sqrt{4\cdot 6}}{2} \Rightarrow x=\dfrac{-2\pm 2\sqrt{6}}{2}$

$\Rightarrow x=\dfrac{-\cancel{2}^{1}\pm\cancel{2}^{1}\sqrt{6}}{\cancel{2}_{1}} \Rightarrow x=\dfrac{-1\pm 1\sqrt{6}}{1} \Rightarrow \boxed{x=-1\pm\sqrt{6}}$

(c) With $x=5$, $x^2-12x+35$ becomes $(5)^2-12\cdot(5)+35 = 25-60+35 = 0$ which does not equal 63. Therefore, $x=5$ is NOT a solution of $x^2-12x+35=63$. F.Y.I. $x=5$ is a solution of $x^2-12x+35=0$.

(d) With $x=-2$, $x^2-12x+35$ becomes $(-2)^2-12\cdot(-2)+35 = 4+24+35 = 63$. Therefore, $x=-2$ is a solution of $x^2-12x+35=63$.

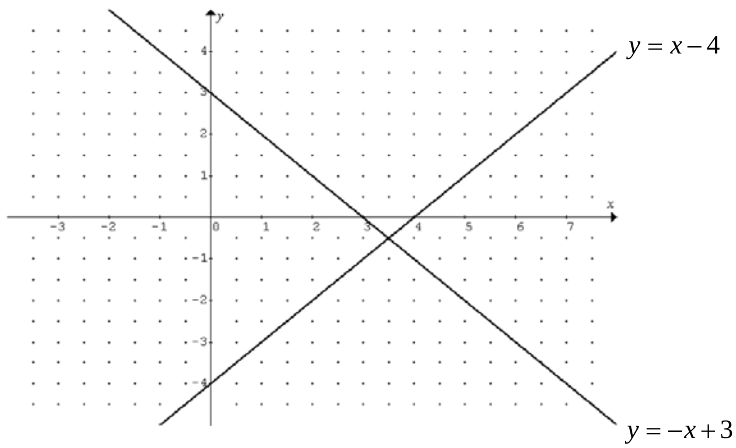
5.

(a) Choosing to eliminate the variable *x* (as opposed to the variable *y*), we see that $\begin{cases}2x+y=3\\4x-7y=2\end{cases}$ is equivalent to $\begin{cases}(-2)\cdot(2x+y)=(3)\cdot(-2)\\4x-7y=2\end{cases} \Rightarrow \begin{cases}-4x-2y=-6\\4x-7y=2\end{cases}$. Adding column-by-column we get $-9y=-4$ which implies that $y=\dfrac{-4}{-9}=\dfrac{4}{9}$. Although one could return to the original system of equations and eliminate *y* (by multiplying both sides of the first equation by 7), I choose to substitute $y=\dfrac{4}{9}$ into one of the original equations. Specifically, I choose to substitute $y=\dfrac{4}{9}$ into the first equation. With $y=\dfrac{4}{9}$, $2x+y=3$ becomes $2x+\dfrac{4}{9}=3$, which implies that $2x=3-\dfrac{4}{9} \Rightarrow 2x=3\cdot\dfrac{9}{9}-\dfrac{4}{9} \Rightarrow 2x=\dfrac{27}{9}-\dfrac{4}{9} \Rightarrow 2x=\dfrac{27-4}{9} \Rightarrow 2x=\dfrac{23}{9} \Rightarrow \dfrac{1}{2}\cdot 2x=\dfrac{23}{9}\cdot\dfrac{1}{2}$

$\Rightarrow x=\dfrac{23\cdot 1}{9\cdot 2} \Rightarrow x=\dfrac{23}{18}$ or $1\frac{5}{18}$. Therefore, the system's solution is $\boxed{(x,y)=\left(\dfrac{23}{18},\dfrac{4}{9}\right)}$.

(b) <u>Choosing</u> to solve the second equation for the variable y (as opposed to solving either of the two equations for either the variable x or the variable y), we see that $2x + y = 8 \quad \Rightarrow \quad y = 8 - 2x$. Substituting the expression $8 - 2x$ in place of y in the first equation, we see that $3x - y = 2$ becomes $3x - (8 - 2x) = 2$. Solving this one-variable equation: $3x - (8 - 2x) = 2 \quad \Rightarrow \quad 3x - 8 + 2x = 2 \quad \Rightarrow \quad 5x - 8 = 2 \quad \Rightarrow \quad 5x = 10 \quad \Rightarrow \quad x = 2$. Thus, with $x = 2$, we see that $y = 8 - 2x$ becomes $y = 8 - 2 \cdot 2 = 8 - 4 = 4$, and so the solution of the system $\begin{cases} 3x - y = 2 \\ 2x + y = 8 \end{cases}$ is $\boxed{(x, y) = (2, 4)}$.

(c) Observe that $\begin{cases} x + y = 3 \\ y = x - 4 \end{cases}$ is equivalent to $\begin{cases} y = -x + 3 \\ y = x - 4 \end{cases}$ (a system in which each of the lines' equations are in *slope-intercept* form, thus making the process for graphing the two lines simple).



From the graph, we see that the two lines intersect at $(x, y) = \left(3\tfrac{1}{2}, -\frac{1}{2}\right) = \left(\frac{7}{2}, -\frac{1}{2}\right)$. To see if $(x, y) = \left(3\tfrac{1}{2}, -\frac{1}{2}\right) = \left(\frac{7}{2}, -\frac{1}{2}\right)$ is the EXACT (correct) solution, one can substitute $x = 3\tfrac{1}{2} = \frac{7}{2}$ and $y = -\frac{1}{2}$ into both of the equations in the original system. With $x = 3\tfrac{1}{2} = \frac{7}{2}$ and $y = -\frac{1}{2}$, we see that each of the equations in $\begin{cases} x + y = 3 \\ y = x - 4 \end{cases}$ is an *identity* since $\begin{cases} 3\tfrac{1}{2} + \left(-\frac{1}{2}\right) = 3 \\ -\frac{1}{2} = 3\tfrac{1}{2} - 4 \end{cases}$. Therefore, the solution of the system $\begin{cases} x + y = 3 \\ y = x - 4 \end{cases}$ is

$\boxed{(x, y) = \left(3\tfrac{1}{2}, -\frac{1}{2}\right) \text{ or } \left(\frac{7}{2}, -\frac{1}{2}\right)}$.

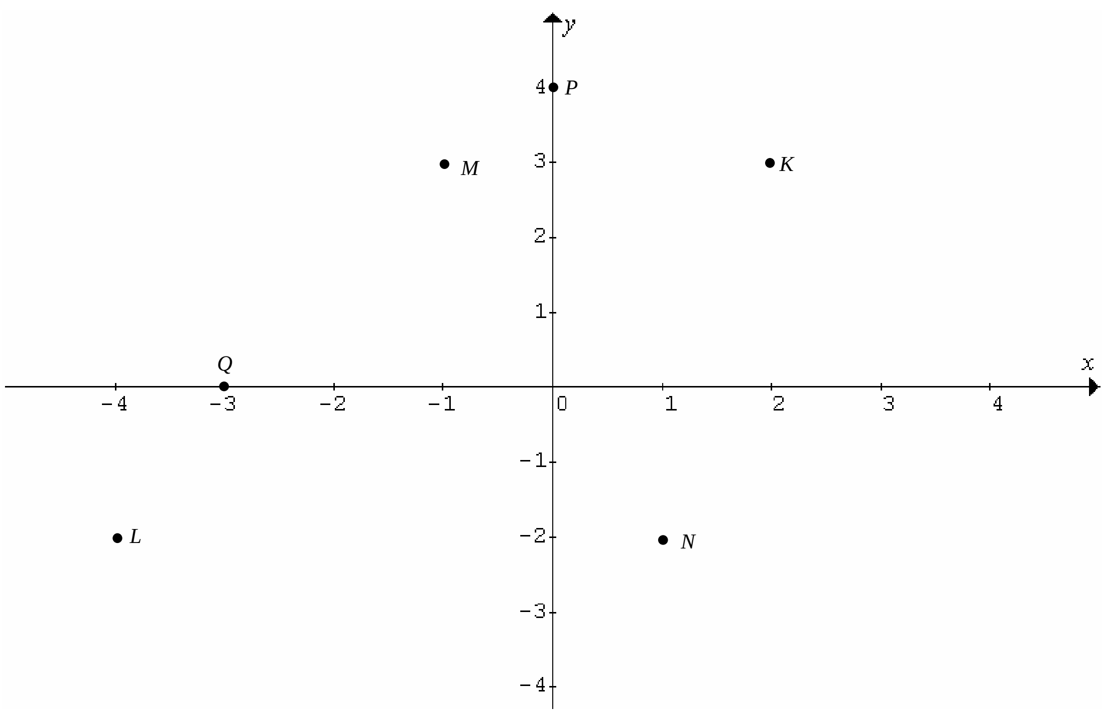
(d) Since a variable, namely *y*, is isolated in one of the two equations in the given system, I choose to employ the method of *Substitution*. Substituting the expression $-\frac{3}{4}x+3$ in place of *y* in the first equation, we see that $3x+4y=12$ becomes $3x+4\left(-\frac{3}{4}x+3\right)=12$. Solving this one-variable equation: $3x+4\left(-\frac{3}{4}x+3\right)=12$ $\Rightarrow$ $3x-3x+12=12$ $\Rightarrow$ $12=12$ (an *identity*). Thus, these two lines are really the SAME, and so they have every point on the line in common. Note that one can double-check this by rewriting the first equation in *slope-intercept* form (by isolating *y*): $3x+4y=12$ $\Rightarrow$ $4y=-3x+12$ $\Rightarrow$ $\frac{4y}{4}=\frac{-3x+12}{4}$ $\Rightarrow$ $y=-\frac{3}{4}x+3$.

Thus, the solutions of the system $\begin{cases} 3x+4y=12 \\ y=-\frac{3}{4}x+3 \end{cases}$ are $\boxed{\text{every point } (x,y) \text{ on the line } y=-\frac{3}{4}x+3}$

(e) Choosing to employ the *Elimination* technique in order to solve the system $\begin{cases} 4x+3y=7 \\ 4x-8=-3y \end{cases}$, we see that the given system is equivalent to $\begin{cases} 4x+3y=7 \\ 4x+3y=8 \end{cases}$. Multiplying both sides of the first equation by -1, we get $\begin{cases} -4x-3y=-7 \\ 4x+3y=8 \end{cases}$. Adding column-by-column, we get $0=1$ (a *contradiction*). Thus, the two lines have no points in common (see for yourself by graphing them), and so we say that the system $\begin{cases} 4x+3y=7 \\ 4x-8=-3y \end{cases}$ has $\boxed{\text{No Solutions}}$.

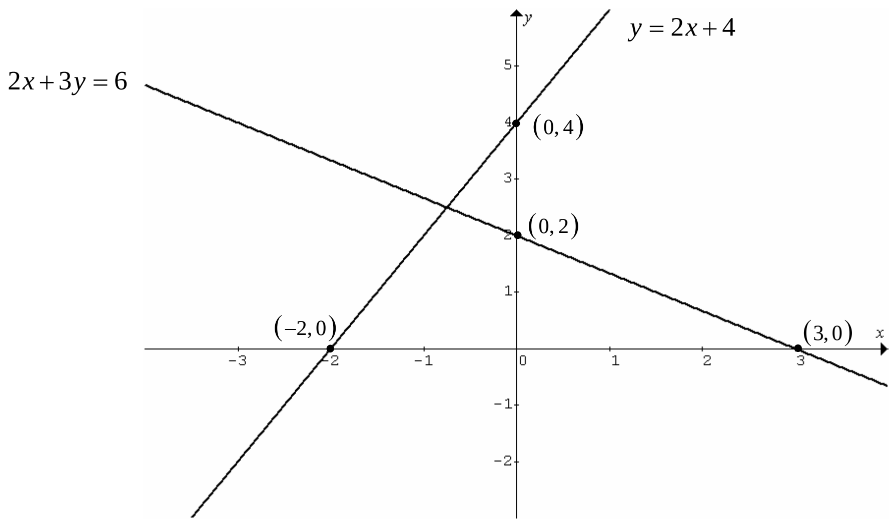
6. Given: $K=(2,3)$, $L=(-4,-2)$, $M=(-1,3)$, $N=(1,-2)$, $P=(0,4)$, $Q=(-3,0)$

(a)



(b) Since $y = 2x + 4$ is in *slope-intercept* form (i.e. it's in $y = mx + b$ form), then the line's *y*-intercept is at $b = 4$. Substituting $y = 0$ in order to determine the *x*-intercept, we have $0 = 2x + 4 \Rightarrow -4 = 2x \Rightarrow -2 = x$. Thus, the *x* and *y* intercepts of the line $y = 2x + 4$ are the points with coordinates $(-2, 0)$ and $(0, 4)$, respectively.

With $x = 0$, we see that $2x + 3y = 6$ becomes $2 \cdot 0 + 3y = 6 \Rightarrow 0 + 3y = 6 \Rightarrow 3y = 6 \Rightarrow y = 2$.
With $y = 0$, we see that $2x + 3y = 6$ becomes $2x + 3 \cdot 0 = 6 \Rightarrow 2x + 0 = 6 \Rightarrow 2x = 6 \Rightarrow x = 3$.
Thus, the *x* and *y* intercepts of the line $2x + 3y = 6$ are the points with coordinates $(3, 0)$ and $(0, 2)$, respectively.



(c) Since $y = 2x + 4$ is in *slope-intercept* form (i.e. it's in $y = mx + b$ form), then this line's slope is $\boxed{m = 2}$.

Recall that the slope can be determined by using the formula $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ if the coordinates of two points on the line are known. Since we know that the points $(3, 0)$ and $(0, 2)$ are on the line $2x + 3y = 6$,

then $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \begin{cases} \frac{2-0}{0-3} = \frac{2}{-3} \\ \frac{0-2}{3-0} = \frac{-2}{3} \end{cases} = -\frac{2}{3}$, and so the slope of this line is $\boxed{m = -\frac{2}{3}}$.

(d) The slope of the line between $K = (2, 3)$ and $L = (-4, -2)$ is $\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \begin{cases} \frac{3-(-2)}{2-(-4)} = \frac{5}{6} \\ \frac{-2-3}{-4-2} = \frac{-5}{-6} \end{cases} = \boxed{\frac{5}{6}}$.

The slope of the line between $M = (-1, 3)$ and $N = (1, -2)$ is $\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \begin{cases} \frac{3-(-2)}{-1-1} = \frac{5}{-2} \\ \frac{-2-3}{1-(-1)} = \frac{-5}{2} \end{cases} = \boxed{-\frac{5}{2}}$.

(e) A line with slope $-\frac{4}{5}$ is a line that is "going down" from left-to-right at the rate of $\frac{4}{5}$. That is, for every 5 units one counts to the right, the line will drop 4 units.

7. RECALL: $y - y_1 = m\left(x - x_1\right)$ can be used to determine the equation of ANY line if the slope and the coordinates of *any* point on the line is known.

$y = mx + b$ can be used to determine the equation of a line if the slope and the *y*-intercept are known.

Recall that the slope can be determined by counting or by using the formula $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

(a) Since the *y*-intercept of the line connecting the points $(2,3)$ and $(4,7)$ is <u>unknown</u>, I choose to use

$y - y_1 = m\left(x - x_1\right)$, where the slope of the line is given by $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \begin{cases} \frac{7-3}{4-2} = \frac{4}{2} \\ \frac{3-7}{2-4} = \frac{-4}{-2} \end{cases} = 2$.

Thus, with $m = 2$ we see that $y - y_1 = m\left(x - x_1\right)$ becomes $y - y_1 = 2\cdot\left(x - x_1\right)$. We can use either point for $\left(x_1\ ,\ y_1\right)$. That is, we can use $(2,3)$ as the known point $\left(x_1\ ,\ y_1\right)$ or we can use $(4,7)$ as the known point $\left(x_1\ ,\ y_1\right)$. Completion of the line's equation is done with both of these below for your edification.

Using $(2,3)$ as the known point $\left(x_1\ ,\ y_1\right)$, then
$y - y_1 = m\left(x - x_1\right)$ becomes $y - 3 = 2\cdot(x-2)$
$\Rightarrow\ y - 3 = 2x - 4\ \Rightarrow\ \boxed{y = 2x - 1}$.

Using $(4,7)$ as the known point $\left(x_1\ ,\ y_1\right)$, then
$y - y_1 = m\left(x - x_1\right)$ becomes $y - 7 = 2\cdot(x-4)$
$\Rightarrow\ y - 7 = 2x - 8\ \Rightarrow\ \boxed{y = 2x - 1}$.

Thus, using either $(2,3)$ or $(4,7)$ as $\left(x_1\ ,\ y_1\right)$, the <u>known point</u> on the line, the slope-intercept form of the line's equation is $y = 2x - 1$. One could choose to change this equation from *slope-intercept* form to g*eneral* form by the following simplification steps: $y = 2x - 1\ \Rightarrow\ 0 = 2x - y - 1\ \Rightarrow\ 1 = 2x - y\ \Rightarrow\ 2x - y = 1$.

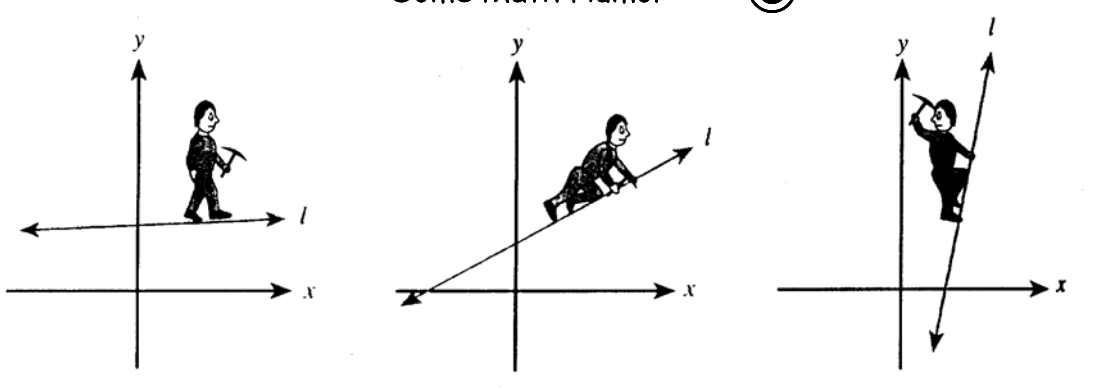
(b) In order to find the equation of the line passing through the point $(4,7)$ with slope -3, I choose to use $y - y_1 = m\left(x - x_1\right)$ (instead of starting with $y = mx + b$ **).** Thus, with $x_1 = 4$, $y_1 = 7$, and $m = -3$, we see that $y - y_1 = m\left(x - x_1\right)$ becomes $y - 7 = -3(x-4)\ \Rightarrow\ y - 7 = -3x + 12\ \Rightarrow\ \boxed{y = -3x + 19}$.

One could choose to change this equation from *slope-intercept* form to g*eneral* form by the following simplification steps: $y = -3x + 19\ \Rightarrow\ 3x + y = 19$.

(c) In the equation written in the form $y = mx + b$, y is the variable whose values are on the vertical axis, x is the variable whose infinitely many values are on the horizontal axis, m represents the line's slope, and b represents the line's y-intercept.

(d) Since $y = 2x + 4$ is in *slope-intercept* form (i.e. it's in $y = mx + b$ form), then this line's slope is $m = 2 = \frac{2}{1}$ and this line's y-intercept is $b = 4$. Thus, a quick method for graphing the line $y = 2x + 4$ is to plot the point with coordiantes $(0,4)$; then move up 2 units and over to the right 1 unit and plot the point with coordinates $(0+1, 4+2) = (1,6)$. Finally, draw a line that passes through both of the points $(0,4)$ and $(1,6)$.

Some Math Humor ☺



8. Don't forget to define your variable(s) very carefully. Each variable should always represent a quantity. Include the appropriate units of measurement

(a) Let x represent the amount of 20% solution (this amount is to be measured in liters).

Let y represent the amount of 50% solution (this amount is to be measured in liters).

Then $\begin{cases} x + y = 10 & \text{[considering the total number of liters]} \\ 0.20x + 0.50y = 0.30 \cdot (10) & \text{[considering the amount of solvent]} \end{cases}$.

$\begin{cases} x + y = 10 \\ 0.20x + 0.50y = 0.30 \cdot (10) \end{cases} \Rightarrow \begin{cases} x + y = 10 \\ 0.2x + 0.5y = 3 \end{cases} \Rightarrow \begin{cases} (-0.2)(x+y) = (10)(-0.2) \\ 0.2x + 0.5y = 3 \end{cases}$ (choosing to eliminate x)

$\Rightarrow \begin{cases} -0.2x - 0.2y = -2 \\ 0.2x + 0.5y = 3 \end{cases}$. Adding column-by-column, we get $0.3y = 1 \Rightarrow y = \frac{1}{0.3} = \frac{1}{\frac{3}{10}} = 1 \cdot \frac{10}{3} = \frac{10}{3} = 3\tfrac{1}{3}$.

Substituting $y = \frac{10}{3} = 3\tfrac{1}{3}$ into the first equation, we see that $x + y = 10$ becomes $x + 3\tfrac{1}{3} = 10 \Rightarrow x = 10 - 3\tfrac{1}{3}$

$\Rightarrow x = 6\tfrac{2}{3}$.

$6\tfrac{2}{3}$ liters of the 20% solution must be mixed with $3\tfrac{1}{3}$ liters of the 50% solution in order to make 10 liters of a 30% solution.

(b) Let d represent the number of dimes.

Let n represent the number of nickels.

Then $\begin{cases} d+n=30 & \text{[considering the total number of coins]} \\ 0.10d+0.05n=2.50 & \text{[considering the value of the coins in dollars]} \end{cases}$.

$\begin{cases} d+n=30 \\ 0.1d+0.05n=2.5 \end{cases} \Rightarrow \begin{cases} d+n=30 \\ (-10)(0.1d+0.05n)=(2.5)(-10) \end{cases}$ (choosing to eliminate d)

$\Rightarrow \begin{cases} d+n=30 \\ -d-0.5n=-25 \end{cases}$. Adding column-by-column, we get $0.5n=5$ which is equivalent to $\frac{1}{2}n=5$, and so $n=2\cdot 5=10$. Substituting $n=10$ into the first equation, we see that $d+n=30$ becomes $d+10=30$ $\Rightarrow d=20$.

Ian has 10 nickels and 20 dimes in his $2.50 collection.

(c) Let B represent the Bill's speed or rate (in kilometers per hour).

Let J represent the Jill's speed or rate (in kilometers per hour).

Then $\begin{cases} J=10+B & \text{[since Jill rides 10 kph faster than Bill]} \\ \frac{10}{B}=\frac{15}{J} & \text{[equating times where } T=\frac{D}{R} \text{ since } D=RT\text{]} \end{cases}$.

$\begin{cases} J=10+B \\ \frac{10}{B}=\frac{15}{J} \end{cases} \Rightarrow \begin{cases} J=10+B \\ 10J=15B \end{cases}$. Substituting the expression $10+B$ for J in the second equation, we get

$10(10+B)=15B \Rightarrow 100+10B=15B \Rightarrow 100=5B \Rightarrow 20=B$. Bill travels at 20 kph.

F.Y.I. With $B=20$, we see that $J=10+B$ becomes $J=10+20=30$ and so Jill travels at 30 kph.

(d) Let x represent the amount of money Mr. Hiyamato invests at 6% solution (this amount is measured in dollars).

Then $8000-x$ represents the amount of money Mr. Hiyamato invests at 7% solution (measured in dollars).

Then, $0.06x+0.07(8000-x)=510 \Rightarrow 0.06x+560-0.07x=510 \Rightarrow -0.01x+560=510$

$\Rightarrow -0.01x=-50 \Rightarrow (-100)(-0.01x)=(-50)(-100) \Rightarrow x=5000$, and so $8000-x=8000-5000=3000$.

Mr. Hiyamato invests $5,000 at 6% and $3,000 at 7%.

(e) Let C represent the cost of the cork (in dollars).

Let B represent the cost of the bottle (in dollars).

Then $\begin{cases} C+B=1.10 \\ C=B-1 \end{cases}$. By substitution, we see that $(B-1)+B=1.10 \Rightarrow 2B-1=1.10 \Rightarrow 2B=2.10$

$\Rightarrow B=\frac{2.10}{2}=1.05$, and so $C=B-1$ becomes $C=1.05-1=0.05$.

The cork costs $0.05 or 5¢ and the cork costs $1.05.

So, if you were to buy 15 corks, each at \$0.05 (or 5¢) and 13 bottles, each at \$1.05, your total cost would be $15(0.05)+13(1.05)=0.75+13.65=14.40$.

The 15 corks and 13 bottles would cost \$14.40.

(f) Let x represent the smaller number. Let y represent the larger number.

Then, $\begin{cases} x+y=14 \\ 2x=y-2 \end{cases} \Rightarrow \begin{cases} x+y=14 \\ 2x+2=y \end{cases}$. Choosing to substitute the expression $2x+2$ in for y in the first equation, we get $x+(2x+2)=14 \Rightarrow 3x+2=14 \Rightarrow 3x=12 \Rightarrow x=4$, and so $y=2x+2$ becomes $y=2(4)+2$ $\Rightarrow y=8+2=10$.

The smaller number is 4 and the larger number is 10.

9.

(a) $(2x^2+3x)+(5x^3+7x^2-31x+4) = 2x^2+3x+5x^3+7x^2-31x+4 = \boxed{5x^3+9x^2-28x+4}$.

(b) $(0.3x^2-7x+4)-(2x^2-0.81x+9) = 0.3x^2-7x+4-2x^2+0.81x-9 = \boxed{-1.7x^2-6.19x-5}$

(c) $\left(\frac{3}{4}y-7\right)\left(4y^2+\frac{5}{7}y\right) = \left(\frac{3}{4}y\right)\cdot(4y^2)+\left(\frac{3}{4}y\right)\cdot\left(\frac{5}{7}y\right)-(7)\cdot(4y^2)-(7)\cdot\left(\frac{5}{7}y\right)$

$= 3y^3+\frac{15}{28}y^2-28y^2-5y = 3y^3+\frac{15}{28}y^2-\frac{28}{1}\cdot\frac{28}{28}y^2-5y = 3y^3+\frac{15}{28}y^2-\frac{784}{28}y^2-5y$

$= \boxed{3y^3-\frac{769}{28}y^2-5y}$.

10.

(a) $\sqrt{25}=\boxed{5}$ **(b)** $\sqrt{36}=\boxed{6}$ **(c)** $\sqrt{125}=\sqrt{25\cdot 5}=\boxed{5\sqrt{5}}$

(d) $d=\sqrt{3^2+5^2}=\sqrt{9+25}=\sqrt{34} \Rightarrow \boxed{d=\sqrt{34}}$

(e) $e^2+2^2=5^2 \Rightarrow e^2+4=25 \Rightarrow e^2=21 \Rightarrow \sqrt{e^2}=\pm\sqrt{21} \Rightarrow e=\pm\sqrt{21}$. However, since e represents a <u>length</u>, it cannot have a negative value, and so $\boxed{e=\sqrt{21}}$.

(f) Let x represent the length of the shorter leg (to be measured in inches).
Then, $x^2+(x+7)^2=17^2 \Rightarrow x^2+(x+7)(x+7)=289 \Rightarrow x^2+x^2+14x+49=289 \Rightarrow 2x^2+14x+49=289$ $\Rightarrow 2x^2+14x-240=0$. Choosing to divide both sides of this equation by 2, we obtain the easier-to-work-with equivalent equation $x^2+7x-120=0$. Since the expression on the left-hand-side of the equation is factorable, I choose not to use the Quadratic Formula to solve $x^2+7x-120=0$ and instead conclude that $x^2+7x-120=0$ $(x+15)(x-8)=0 \Rightarrow$ Either $x+15=0 \Rightarrow x=-15$ (we discard this solution as a possible value for <u>length</u> x) or $x-8=0 \Rightarrow x=8$. Thus, one leg of this triangle is 8 inches long and the other is 15 inches long (since it was 7 inches longer than x).

The lengths of the triangle's legs are 8 inches and 15 inches.