PROBLEMS USING COMBINATIONS AND PROBABILITY

License Plate Problems

Each of the following problems assumes that a state's license plates consist of a certain number of letters followed by a certain number of numbers.

1) How many different plates can be made with one letter followed by one number?

There are 26 choices of letters. Each of these 26 letters can be followed by one of 10 different digits -- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Therefore there are 26 x 10 = 260 different plates which can be made.

2) How many different plates can be made with two letters followed by three numbers?

There are 26 choices for the first letter. For each of these letters, there are 26 choices for the second letter. There are therefore 26X26 = 676 possible pairs of letters. (Note that a repeat letter, such as DD, is allowed and so we do not use ${}_{26}P_2$.)

You must now consider the three numbers. There are 10 possibilities for the first digit, 10 possibilities for the second, and 10 possibilities for the third. This means that there are 10 x 10 x10 = 1000 different numbers. (Note that this is simply saying that there are 1000 numbers between and including 000 and 999.)

Combining these results, it follows that there are 676 x 1000 = 676,000 different license plates possible.

3) What is the probability that I will have a license plate with my initials, MR, followed by a number which ends in 4?

The probability of any event occurring is (the number of ways the event can occur) divided by (the number of possible outcomes). Since MR can occur in only one way and there are 676 possible pairs of letters, the probability of getting MR is $\frac{1}{676}$. There are ten different choices for the last digit, but only one of these choices is a 4. Therefore the probability of getting a number which ends in 4 is $\frac{1}{10}$.

Selecting the letters and the numbers are independent events, and so the probability of receiving a license plate which has the letters MR followed by a number which ends in 4 is $\frac{1}{676} \times \frac{1}{10} = \frac{1}{6760}$.