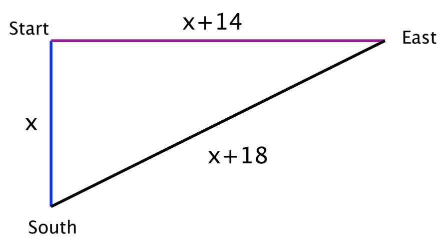
Math 99 Worksheet #6 Solutions

1. Two cars left an intersection at the same time. One traveled south and the other traveled east. After a certain amount of time, the eastbound car has traveled 14 miles further than the southbound car. How far has each car traveled if the distance between them was 4 miles more than the distance the east bound car has traveled?

   Here is a couple of ways to set up the equations:

   - Let $x =$ Distance traveled by the southbound car
     $\Rightarrow$ $x + 14 =$ Distance traveled by the eastbound car
     $x + 18 =$ Distance between the cars

     We have the following diagram:

     Equation: $x^2 + (x + 14)^2 = (x + 18)^2$

     $$x^2 + x^2 + 28x + 196 = x^2 + 36x + 324$$

     $$x^2 - 8x - 128 = 0$$

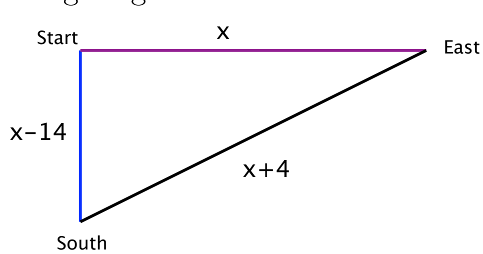
     You can factor this into $(x - 16)(x + 8) = 0$ and get the solutions $x = 16$ and $x = -8$ or you can use the quadratic formula to obtain the same solutions.

     Only the solution of $x = 16$ makes sense in our situation since distances should be positive. So, the southbound car travels 16 miles, the eastbound car travels 30 miles, and the distance between the cars is 34 miles.

     Check: $16^2 + 30^2 = 34^2$ ✓

   - Let $x =$ Distance traveled by the eastbound car
     $\Rightarrow$ $x - 14 =$ Distance traveled by the southbound car
     $x + 4 =$ Distance between the cars

     We have the following diagram:

Equation: $$(x-14)^2 + x^2 = (x+4)^2$$
$$x^2 - 28x + 196 + x^2 = x^2 + 8x + 16$$
$$x^2 - 36x + 180 = 0$$

You can factor this into $(x-6)(x-30) = 0$ and get the solutions $x = 6$ and $x = 30$ or you can use the quadratic formula to obtain the same solutions.

Only the solution of $x = 30$ makes sense in our situation since distances should be positive and $x = 6$, then the southbound car will have traveled -8 miles, which does not make sense in our situation. So, the southbound car travels 16 miles, the eastbound car travels 30 miles, and the distance between the cars is 34 miles.

Check: $16^2 + 30^2 = 34^2$ ✓

2. A ball is thrown downward from the top of a 180-foot building with an initial velocity of 24 feet per second. The height of the ball in feet after t seconds is given by $h = -16t^2 - 20t + 180$.

   (a) What is the height of the ball after 1 second?

   We are looking for the value of h when $t = 1$.
   Plugging in $t = 1$: $h = -16(1)^2 - 20(1) + 180 = 144$

   So, the ball is 144 feet above the ground at 1 second.

   (b) How long does it take for the ball to strike the ground? (Round to the nearest tenth of a second.)

   We are looking for the time ($t = ?$) when the ball lands or when $h = 0$.
   Plugging in $h = 0$: $0 = -16t^2 - 20t + 180$

   Solving for t: You can use the quadratic formula to solve for t in the equation above as it is or you can divide all of the values by -4 first.

   $$\Rightarrow \quad 0 = 4t^2 + 5t - 45 \quad \text{(Division by } -4\text{)}$$
   Using the quadratic formula with $a = 4$, $b = 5$, and $c = -45$:
   $$t = \frac{-5 \pm \sqrt{5^2 - 4(4)(-45)}}{2(4)} = \frac{-5 \pm \sqrt{25+720}}{8} = \frac{-5 \pm \sqrt{745}}{8} \approx 2.78683 \quad \text{or} \quad -4.0368$$

   Since time must be positive, the solutions must be that $t \approx 2.8$. So, the ball lands at approximately 2.8 seconds.

   (c) When will the ball be 90 feet from the ground? (Round to the nearest tenth of a second.)

We are looking for the time ($t = ?$) when $h = 90$.
Plugging in $h = 90$: $\quad 90 = -16t^2 - 20t + 180 \quad \Rightarrow \quad 0 = -16t^2 - 20t + 90$

Solving for t: You can use the quadratic formula to solve for t in the equation above as it is or you can divide all of the values by -2 first.

$$\Rightarrow \quad 0 = 8t^2 + 10t - 45 \quad \text{(Division by } -4\text{)}$$

Using the quadratic formula with $a = 8$, $b = 10$, and $c = -45$:

$$t = \frac{-10 \pm \sqrt{10^2 - 4(8)(-45)}}{2(8)} = \frac{-10 \pm \sqrt{100 + 1440}}{16}$$
$$= \frac{-10 \pm \sqrt{1540}}{16}$$
$$\approx 1.827677 \quad \text{or} \quad -3.077677$$

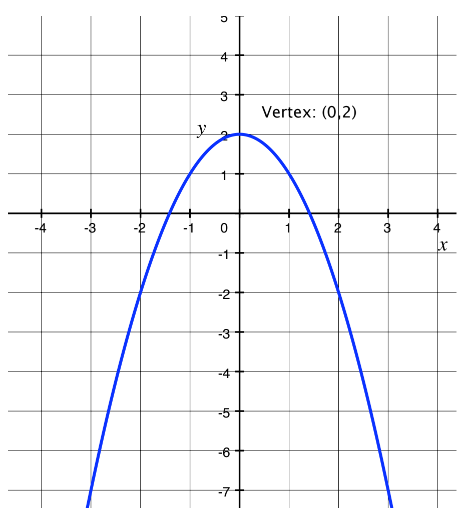
Since time must be positive, the solutions must be that $t \approx 1.8$. So, the ball is 90 feet high at approximately 1.8 seconds.

3. Graph the following parabolas and state the domain, range, and vertex for each graph.

   (a) $f(x) = -x^2 + 2$

   This parabola will be opening downward (since there is a negative sign in front of the x^2) and it will be shifted up by 2 units.

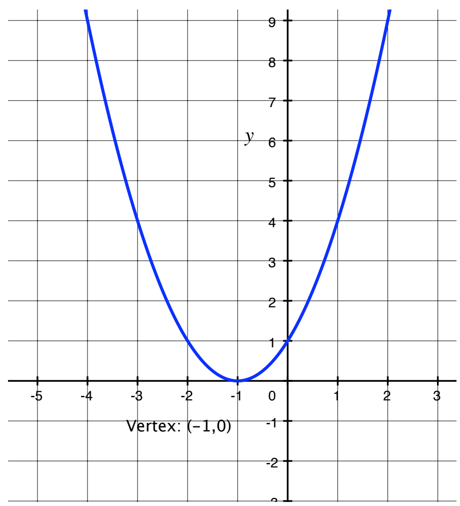
x	y
-2	$-(-2)^2 + 2 = -2$
-1	$-(-1)^2 + 2 = 1$
0	2
1	$-1^2 + 2 = 1$
2	$-2^2 + 2 = 2$



(b) $g(x) = (x+1)^2$

This parabola will be opening upward and it will be shifted left by 1 unit.

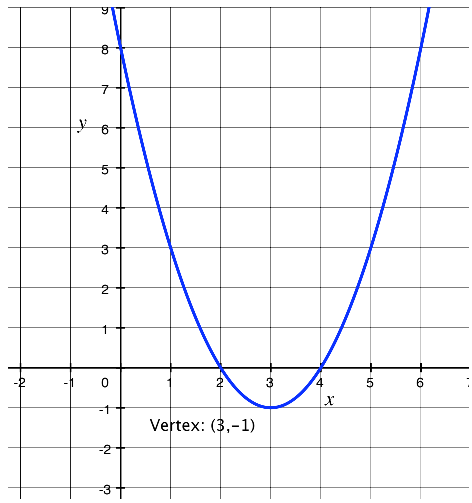
x	y
-3	$(-3+1)^2 = 4$
-2	$(-2+1)^2 = 1$
-1	$(-1+1)^2 = 0$
0	$(0+1)^2 = 1$
1	$(1+1)^2 = 4$



(c) $h(x) = (x-3)^2 - 1$

This parabola will be opening upward. It will be shifted right by 3 units and down by 1 unit.

x	y
1	$(1-3)^2 - 1 = 3$
2	$(2-3)^2 - 1 = 0$
3	$(3-3)^2 - 1 = -1$
4	$(4-3)^2 - 1 = 0$
5	$(5-3)^2 - 1 = 3$



Vertex: (3,−1)
x
y