Math 99 Worksheet #3 Solutions

1. Solve the following compound inequalities.

   (a) $3x - 3 < 7 + x \quad \text{and} \quad 5 - x \geq 2$

   Solving both inequalities:

$$\begin{array}{rcr} 3x - 3 < 7 + x & \text{and} & 5 - x \geq 2 \\ 2x < 10 & \text{and} & -x \geq -3 \\ x < 5 & \text{and} & x \leq 3 \end{array}$$

   The solution set will be the intersection (and) of the two sets. $\Rightarrow \quad (-\infty, 3]$

   (b) $-2y + 6 > 4 \quad \text{or} \quad y - 1 < 11$

   Solving both inequalities:

$$\begin{array}{rcr} -2y + 6 > 4 & \text{or} & y - 1 < 11 \\ -2y > -2 & \text{or} & y < 12 \\ y < 1 & \text{or} & y < 12 \end{array}$$

   The solution set will be the union (or) of the two sets. $\Rightarrow \quad (-\infty, 12)$

   (c) $t + 8 \geq 11 \quad \text{and} \quad 3t - 4 < 2$

   Solving both inequalities:

$$\begin{array}{rcr} t + 8 \geq 11 & \text{and} & 3t - 4 < 2 \\ t \geq 3 & \text{and} & 3t < 6 \\ t \geq 3 & \text{and} & t < 2 \end{array}$$

   The solution set will be the intersection (and) of the two sets. $\Rightarrow \quad \emptyset$ (No solution)

   (d) $3m - 4 > 5m + 6 \quad \text{or} \quad m > -7$

   Solving both inequalities:

$$\begin{array}{rcr} 3m - 4 > 5m + 6 & \text{or} & m > -7 \\ -10 > 2m & \text{or} & m > -7 \\ -5 > m & \text{or} & m > -7 \end{array}$$

   The solution set will be the union (or) of the two sets. $\Rightarrow \quad (-\infty, \infty)$ or $\mathbb{R}$

2. Solve the following absolute value equations and inequalities.

(a) $|4x - 5| = 17$

We have two possibilities:

$$\begin{array}{rcr} 4x - 5 = 17 & \text{or} & 4x - 5 = -17 \\ 4x = 22 & \text{or} & 4x = -12 \\ x = \frac{22}{4} = \frac{11}{2} & \text{or} & x = -3 \end{array}$$

So, the solution set is $\{-3, \frac{11}{2}\}$.

(b) $|3t + 1| > 25$

We have two possibilities:

$$\begin{array}{rcr} 3t + 1 > 25 & \text{or} & 3t + 1 < -25 \\ 3t > 24 & \text{or} & 3t < -26 \\ t > 8 & \text{or} & t < -\frac{26}{3} \end{array}$$

Solution Set: $(-\infty, -\frac{26}{3}) \cup (8, \infty)$

(c) $| - 2r - 4| \leq 8$

To satisfy the absolute value, we must have

$$\begin{array}{rl} -8 \leq -2r - 4 \leq 8 & \\ -4 \leq -2r \leq 12 & \text{(Adding 4 to all parts)} \\ 2 \geq r \geq -6 & \text{(Dividing all parts by } -2\text{)} \end{array}$$

Solution Set: $[-6, 2]$

(d) $|3s + 2| \geq -2$

Note that $|3s + 2| \geq 0$ for all values of s since the absolute value is always positive or 0. So, the absolute value inequality $|3s + 2| \geq -2$ is satisfied by any value of s.

Solution Set: $(-\infty, \infty)$ or $\mathbb{R}$