Math 99 Worksheet #1 Solutions

1. Is $(-1,-3)$ a solution to the system $\begin{array}{l} 3x+5y=-18 \\ 4x+2y=10 \end{array}$?

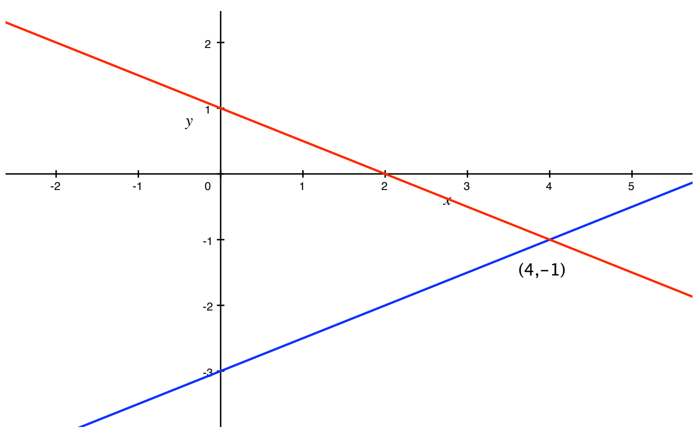
   Plugging in the values $x=-1$ and $y=-3$ into the first equation:
   $3(-1)+5(-3)=-3-15=-18 \quad \Rightarrow \quad (-1,-3)$ is a solution to equation 1

   Plugging in the values $x=-1$ and $y=-3$ into the second equation:
   $4(-1)+2(-3)=-4-6=-10 \neq 10 \quad \Rightarrow \quad (-1,-3)$ is not a solution to equation 2

   So, $(-1,-3)$ is not a solution to the system.

2. Solve the system $\begin{array}{l} 2x-4y=12 \\ x+2y=2 \end{array}$ by graphing.

   Rewriting the equations in slope-intercept form: $\begin{array}{l} y=\frac{1}{2}x-3 \\ y=-\frac{1}{2}x+1 \end{array}$



   The point of intersection is $(4,-1)$ so our solution to the system is $(4,-1)$.
   Check: First equation $2(4)-4(-1)=8+4=12$
   Second equation $4+2(-1)=4-2=2$

3. Without graphing, determine whether the following systems have one solution, no solution, or an infinite number of solutions.

   (a) $\begin{array}{l} y=-\frac{1}{2}x+3 \\ y=5x-4 \end{array}$

   (b) $\begin{array}{l} y=-3x+4 \\ 6x-2y=-4 \end{array}$

For part (a), the slope of the first line is $-\frac{1}{2}$ and the slope of the second line is 5. Since the slopes are different, the lines must intersect at one point . $\Rightarrow$ The system has one solution.

For part (b), the slope of the first line is -3 and the slope of the second line is 3 (slope-intercept form $y = 3x+2$). Since the slope are different, the lines must intersect at one point. $\Rightarrow$ The system has one solution.

4. Solve each system below by substitution.

   (a) $x + 3y = -28$
   $y = 2x$

   (b) $2x = -12 + y$
   $2y = 4x + 24$

   (c) $\frac{1}{2}x = \frac{1}{2}y + 2$
   $\frac{1}{2}x + \frac{1}{3}y = \frac{9}{2}$

   - For part (a), the second equation is already solved for y.
     Substitution: $x + 3(2x) = -28 \quad \Rightarrow \quad x + 6x = -28 \quad \Rightarrow \quad 7x = -28 \quad \Rightarrow \quad x = -4$

     Solving for y: $y = 2(-4) = -8$
     $\Rightarrow$ Solution: $(-4, -8)$

     Check: First equation $-4 + 3(-8) = -4 - 24 = -28$
     Second equation $-8 = 2(-4)$

   - For part (b), solving the first equation for y: $y = 2x + 12$
     Substitution: $2(2x + 12) = 4x + 24 \quad \Rightarrow \quad 2x + 24 = 4x + 24 \quad \Rightarrow \quad 0 = 0$

     This implies that the two equations describe the same line. So, we have an infinite number of solutions. The solution set is $\{(x, y)|2x = -12 + y\}$.

   - For part (c), we should clear the fractions first to make the equations easier to work with.
     Multiplying the first equation by 2 (on both sides) and the second equation by 6 (on both sides) yields the new system $x = y + 4$
     $3x + 2y = 27$

     The first equation is already solved for x.

     Substitution: $3(y+4)+2y = 27 \quad \Rightarrow \quad 3y+12+2y = 27 \quad \Rightarrow \quad 5y = 15 \quad \Rightarrow \quad y = 3$

     Solving for y: $x = 3 + 4 = 7$
     $\Rightarrow$ Solution: $(7, 3)$

     Check: First equation $\frac{1}{2}(7) = \frac{1}{2}(3) + 2$
     Second equation $\frac{1}{2}(7) + \frac{1}{3}(3) = \frac{9}{2}$

5. Solve each system below by elimination.

(a) $x - y = -2$
$x + y = 8$

(b) $6x - y = -1$
$5y = 17 + 6x$

(c) $3x - 5y = 1$
$6x - 10y = 4$

- For part (a), since the y terms in each equation have coefficients that are opposite in sign, we can add the equations and eliminate y.

$$\begin{array}{l} x - y = -2 \\ \underline{x + y = 8} \\ 2x \quad\ \ = 6 \quad \Rightarrow \quad x = 3 \end{array}$$

Solving for y using the second equation (could also use the first equation):

$$3 + y = 8 \quad \Rightarrow \quad y = 5$$
$$\text{Solution: } (3, 5)$$

Check: First equation $3 - 5 = -2$
Second equation $3 + 5 = 8$

- For part (b), we must first put the equations in standard form.

Standard form: $6x - y = -1$
$-6x + 5y = 17$

Since the x terms in each equation have coefficients that are opposite in sign, we can add the equations and eliminate x.

$$\begin{array}{r} 6x - y = -1 \\ \underline{-6x + 5y = 17} \\ 4y = 16 \end{array} \quad \Rightarrow \quad y = 4$$

Solving for x using the first equation (could also use the second equation):

$$6x - 4 = -1 \quad \Rightarrow \quad 6x = 3 \quad \Rightarrow \quad x = \tfrac{1}{2}$$
$$\text{Solution: } (\tfrac{1}{2}, 4)$$

Check: First equation $6(\frac{1}{2}) - 4 = -1$
Second equation $-6(\frac{1}{2}) + 5(4) = 17$

- For part (c), we must multiply by constants so that we can eliminate a variable.

Multiplying the first equation by -2 gives us the new system:
$-6x + 10y = -2$

$$6x - 10y = 4$$

Adding the two equations:

$$\begin{array}{r} -6x + 10y = -2 \\ \underline{6x - 10y = 4} \\ 0 = 2 \end{array}$$

This false statement indicates that there is no solution to the system. (Parallel lines) The solution set is $\emptyset$.