Math 99
Exam 3 Solutions

1. (10 pts.) Carry out the following operations and simplify as much as possible.

   (a) (5 pts.) $(5-3i)-(2+i)$

   $$(5-3i)-(2+i) \;=\; 5-3i-2-i \;=\; 3-4i$$

   (b) (5 pts.) $(5-3i)\cdot(2+i)$

   $$\begin{aligned}(5-3i)\cdot(2+i) &= 10+5i-6i-3i^2 \quad &&\text{(FOIL)}\\ &= 10-i-3(-1) &&(i^2=-1)\\ &= 13-i\end{aligned}$$

2. (25 pts.) You launch a toy rocket with an initial velocity of 120 feet/second. The height of the rocket is given by $h=-16t^2+120t$ in feet at t seconds.

   (a) (10 pts.) When does the rocket land?

   When the rocket lands, the height of the rocket will be 0 feet, so we are trying to find a value of t so that $h=0$.

   Setting $h=0$ and solving for t:
   $$\begin{aligned}0 &= -16t^2+120t\\ 0 &= -8t(2t-15) \quad \text{(Factoring out } -8t\text{)}\end{aligned}$$

   By the zero-factor property, we have that either $-8t=0$ or $2t-15=0$.
   So, either $t=0$ seconds or $t=7.5$ seconds.

   Since the rocket launches at 0 seconds, it must land at 7.5 seconds.

   (b) (5 pts.) How high is the rocket after 3 seconds?

   We are looking for the height ($h=?$) when $t=3$.

   Setting $t=3$ and solving for h:
   $$\begin{aligned}h &= -16(3)^2+120(3)\\ &= -16(9)+120(3) = 216 \text{ feet}\end{aligned}$$

   So, the height is 216 feet at 3 seconds.

   (c) (10 pts.) Find all the times at which the rocket is 144 feet high.

   We are looking for the time ($t=?$) at which $h=144$.

   Setting $h=144$ and solving for t:
   $$\begin{aligned}144 &= -16t^2+120t\\ 0 &= -16t^2+120t-144\end{aligned}$$

$$0 = 2t^2 - 15t + 18 \quad \text{(Dividing both sides by } -8\text{)}$$

Solving the equation (Here are two ways):

- Factoring: $0 = (2t-3)(t-6)$

$$\Rightarrow \quad 2t - 3 = 0 \quad \text{or} \quad t - 6 = 0$$
$$\Rightarrow \quad t = 1.5 \quad \text{or} \quad t = 6$$

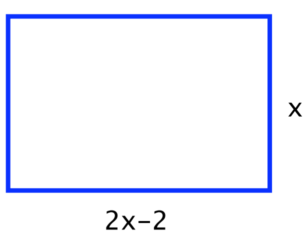
So, the rocket is 144 feet high at 1.5 seconds and 6 seconds.

- Quadratic Formula: $a = 2,\ b = -15,\ c = 18$

$$t = \frac{15 \pm \sqrt{(-15)^2 - 4(2)(18)}}{2(2)} = \frac{15 \pm \sqrt{225-144}}{4} = \frac{15 \pm \sqrt{81}}{4}$$
$$= \frac{15 \pm 9}{4}$$
$$= \frac{24}{4} \text{ or } \frac{6}{4}$$
$$= 6 \text{ or } 1.5$$

So, the rocket is 144 feet hight at 1.5 seconds and 6 seconds.

3. (15 pts.) The following rectangle has an area of 84 square units. What are the dimensions of the rectangle given the lengths in the diagram below?



Width of rectangle $= x$
Length of rectangle $= 2x - 2$
$\Rightarrow$ Area of rectangle $= x(2x-2)$

$$\Rightarrow \quad 84 = x(2x-2)$$
$$84 = 2x^2 - 2x$$
$$0 = 2x^2 - 2x - 84$$
$$0 = x^2 - x - 42$$

Factoring this equation: $0 = (x-7)(x+6)$

$$\Rightarrow \quad x - 7 = 0 \quad \text{or} \quad x + 6 = 0$$
$$\Rightarrow \quad x = 7 \quad \text{or} \quad x = -6$$

Since lengths are positive, the only solution that makes sense in this application is $x = 7$. If $x = 7$, then the dimensions of the rectangle are 7 by 12 units.

4. (15 pts.)

   (a) (5 pts.) Use the discriminant to determine the number and type (rational, irrational, or imaginary) of solutions of the equation $x^2 - 6x + 13 = 0$.

   Discriminant $= b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16 < 0$

Since the discriminant is negative, we have 2 imaginary solutions to the equation.

(b) (10 pts.) Use the quadratic formula to solve the equation $x^2 - 6x + 13 = 0$.

Solutions: $$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

5. (20 pts.) Solve the following equations. Simplify your answer as much as possible.

(a) (10 pts.) $(3m+6)^2 + 4 = 1$

$(3m+6)^2 + 4 = 1 \quad \Rightarrow \quad (3m+6)^2 = -3$

Using the square root property, we have that $3m + 6 = \pm\sqrt{-3} = \pm i\sqrt{3}$

$$\Rightarrow \quad 3m = -6 \pm i\sqrt{3}$$
$$\Rightarrow \quad m = \frac{-6 \pm i\sqrt{3}}{3}$$

(b) (10 pts.) $4x^2 + 22x = 12$ (Solve by completing the square.)

$x^2 + \frac{11}{2}x = 3$ (Dividing both sides by 4)

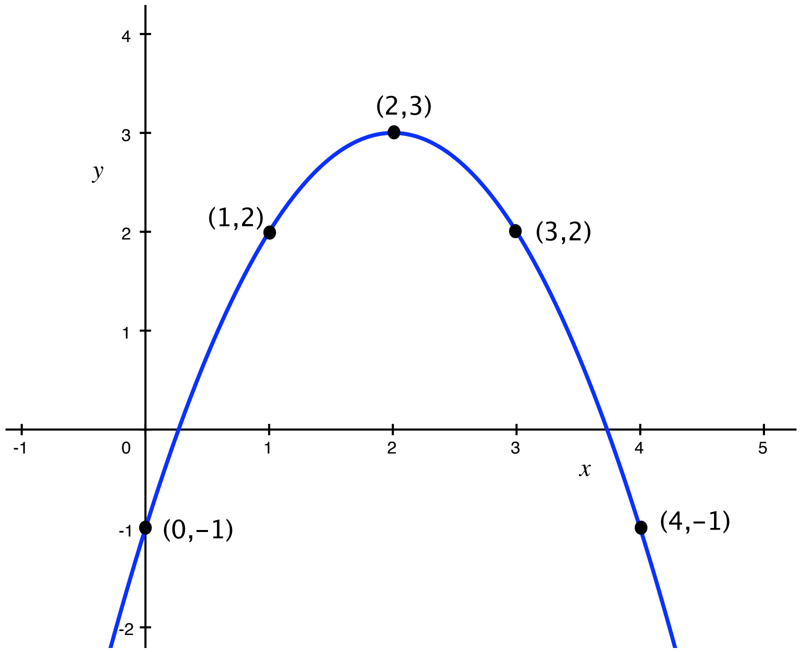
Note that $(x + \frac{11}{4})^2 = x^2 + \frac{11}{2}x + (\frac{11}{4})^2 = x^2 + \frac{11}{2}x + \frac{121}{16}$.

Adding $\frac{121}{16}$ to both sides of the equation:

$$x^2 + \frac{11}{2}x + \frac{121}{16} = 3 + \frac{121}{16} = \frac{48}{16} + \frac{121}{16} = \frac{169}{16}$$

$$(x + \tfrac{11}{4})^2 = \frac{169}{16}$$
$$x + \frac{11}{4} = \pm\sqrt{\frac{169}{16}} = \pm\frac{13}{4}$$
$$x = -\frac{11}{4} \pm \frac{13}{4} = \frac{1}{2} \quad \text{or} \quad -6$$

6. (15 pts.) Graph the parabola $f(x) = -(x-2)^2 + 3$ on the axis below. Plot at least 2 points in addition to the vertex. State the vertex, domain, and range.



Here is the graph including various points on the graph in addition to the vertex.

Vertex: $(2, 3)$
Domain: $(-\infty, \infty)$ or all real numbers
Range: $(\infty, 3]$