Math 99
Exam 2 Solutions

1. (15 pts.) $f(x) = \frac{3}{2x-8}$

   (a) (5 pts.) What is the domain of f?

   The function $f(x)$ will not be defined when the denominator of f equals zero. Since the denominator is equal to zero only when $x = 4$, the domain of f is all real numbers except $x = 4$ or $(-\infty, 4) \cup (4, \infty)$.

   (b) (5 pts.) What is $f(-1)$?

   $f(-1) = \frac{3}{2(-1)-8} = \frac{3}{-10} = -\frac{3}{10}$

   (c) (5 pts.) What is $f(3-x)$? (Simplify your answer as much as possible.)

   $f(3-x) = \frac{3}{2(3-x)-8} = \frac{3}{6-2x-8} = \frac{3}{-2x-2}$

2. (10 pts.) Find the distance between the points $(\sqrt{6}, 2)$ and $(-\sqrt{6}, -1)$.

   Using the distance formula, we have that the distance is

$$\begin{aligned}\text{Distance} = \sqrt{(-\sqrt{6}-\sqrt{6})^2 + (-1-2)^2} &= \sqrt{(-2\sqrt{6})^2 + (-3)^2} \\ &= \sqrt{4 \cdot 6 + 9} \\ &= \sqrt{33}\end{aligned}$$

3. (20 pts.) Solve the following equations.

   (a) (10 pts.) $2\sqrt[3]{3x+1} + 4 = 0$

   Isolating the radical:
$$\begin{aligned}2\sqrt[3]{3x+1} &= -4 \\ \sqrt[3]{3x+1} &= -2\end{aligned}$$

   Cubing both sides:
$$3x + 1 = -8$$
$$\Rightarrow \quad 3x = -9 \quad \Rightarrow \quad x = -3$$

$$\Rightarrow \quad x = -3 \text{ is a possible solution to the original equation.}$$

   Check: $2\sqrt[3]{3(-3)+1} + 4 = 2\sqrt[3]{-8} + 4 = -4 + 4 = 0$ ✓

   So, $x = -3$ is a solution.

   (b) (10 pts.) $\sqrt{5+t} = t + 3$

$$\begin{aligned} \text{Squaring both sides:} \quad & 5+t=(t+3)^2=(t+3)(t+3) \\ \Rightarrow \quad & 5+t=t^2+6t+9 \\ & 0=t^2+5t+4 \\ & 0=(t+4)(t+1) \end{aligned}$$

$\Rightarrow \quad t=-4$ or $t=-1$ are possible solutions.

Check: $\underline{t=-4}$ $\quad \sqrt{5-4} \neq -4+3$

$\underline{t=-1}$ $\quad \sqrt{5-1}=-1+3$ ✓

So, $t=-1$ is the solution of the original equation.

4. (40 pts.) Simplify the following as much as possible.

   (a) (6 pts.) $\sqrt{\frac{m^6}{81}}=\frac{m^3}{9}$

   (b) (7 pts.) $2a^{2/3}(a^{4/3}-3a^{-5/3})$

$$\begin{aligned} 2a^{2/3}(a^{4/3}-3a^{-5/3}) &= 2a^{2/3}\cdot a^{4/3}-6a^{2/3}\cdot a^{-5/3} \\ &= 2a^{6/3}-6a^{-3/3} \\ &= 2a^2-6a^{-1} \\ &= 2a^2-\frac{6}{a} \end{aligned}$$

   (c) (6 pts.) $\sqrt[4]{r^3}\cdot\sqrt[8]{r^3}$ (Leave your answer in exponential form.)

$$\sqrt[4]{r^3}\cdot\sqrt[8]{r^3}=r^{3/4}\cdot r^{3/8}=r^{6/8}\cdot r^{3/8}=r^{9/8}$$

   (d) (7 pts.) $\sqrt[3]{16x^6y^{11}}=2x^2y^3\sqrt[3]{2y^2}$

   (e) (7 pts.) $$\begin{aligned} 4-\sqrt{6}+3\sqrt{54} &= 4-\sqrt{6}+3\sqrt{9\cdot 6} \\ &= 4-\sqrt{6}+9\sqrt{6} \\ &= 4+8\sqrt{6} \end{aligned}$$

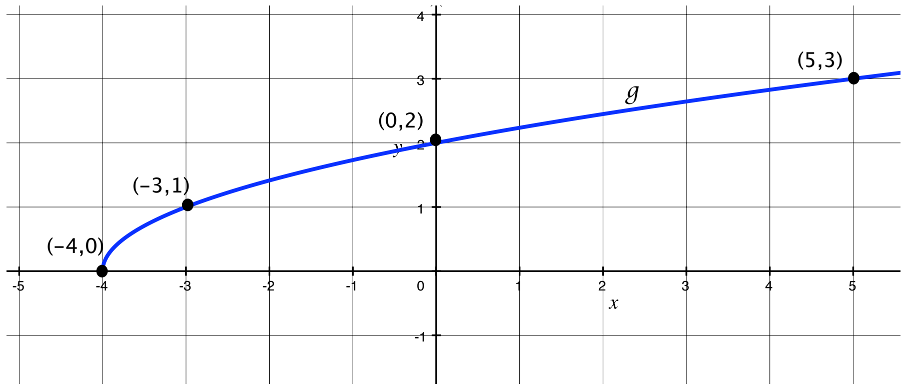
   (f) (7 pts.) $\frac{\sqrt{27}}{2+\sqrt{3}}$ (Rationalize the denominator and simplify.)

$$\frac{\sqrt{27}}{2+\sqrt{3}}=\frac{3\sqrt{3}}{2+\sqrt{3}}=\frac{3\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{3\sqrt{3}(2-\sqrt{3})}{4-3}=3\sqrt{3}(2-\sqrt{3})=6\sqrt{3}-3\cdot 3=6\sqrt{3}-9$$

5. (15 pts.) Graph $g(x)=\sqrt{x+4}$ and state the domain and range of g.

   Finding points on the graph:

x	y
-4	$\sqrt{-4+4} = 0$
-3	$\sqrt{-3+4} = 1$
0	$\sqrt{0+4} = 2$
5	$\sqrt{5+4} = 3$



Domain: $[-4, \infty)$ or $x \geq -4$ **Range:** $[0, \infty)$ or $y \geq 0$