Math 99
Exam 1 Solutions

1. (20 pts.) Rocky sells 2 types of pet rocks. He sells basic pet rocks for \$5.10 each and premium pet rocks with glitter for \$12.25 each. On a particular day, his sales were \$335.75 and he sold 8 more basic pet rocks than premium pet rocks. Find how many of each type he sold on that day by writing a linear system that models the situation.

Unknowns: Number of basic pet rocks sold $= b$
Number of premium pet rocks sold $= p$

Note: Total value of basic pet rocks sold $= 5.10b$ (Total Value = price $\times$ quantity)
Total value of premium pet rocks sold $= 12.25p$

System of equations: $b = p + 8$
$5.1b + 12.25p = 335.75$

Using substitution, we have $5.1(p+8) + 12.25p = 335.75$

$$\begin{aligned} &\Rightarrow \quad 5.1p + 40.8 + 12.25p = 335.75 \\ &\Rightarrow \quad 17.35p = 294.95 \\ &\Rightarrow \quad p = 17 \end{aligned}$$

Solving for b: $b = 17 + 8 = 25$

So, Rocky sold 25 basic pet rocks and 17 premium pet rocks.

2. (5 pts.) Is $(0, 1, 2)$ a solution to the system $\begin{aligned} 3x + y - z &= -1 \\ -x - 2y + 2z &= 2 \\ 4x + 5y - z &= 3 \end{aligned}$?

Plugging $x = 0,\ y = 1,\ z = 2$ into each equation:
Equation 1: $3(0) + 1 - 2 = -1$ ✓
Equation 1: $0 - 2(1) + 2(2) = 2$ ✓
Equation 1: $4(0) + 5(1) - 2 = 3$ ✓

Since $(0, 1, 2)$ satisfies all three equations, it is a solution to the system.

3. (20 pts.) Solve the following absolute value equations and inequalities. Write your solutions in interval notation (as simplified as possible).

(a) (6 pts.) $|\frac{1}{2}x - 3| - 4 = 1$

Adding 4 to both sides: $|\frac{1}{2}x - 3| = 5$

To satisfy the absolute value equation, we must have

$$\begin{array}{ccc} \frac{1}{2}x - 3 = 5 & \text{or} & \frac{1}{2}x - 3 = -5 \\ \frac{1}{2}x = 8 & \text{or} & \frac{1}{2}x = -2 \\ x = 16 & \text{or} & x = -4 \end{array}$$

Solution set: $\{-4, 16\}$

(b) (7 pts.) $|r + 1| \geq 3$

To satisfy the absolute value inequality, we must have

$$\begin{array}{ccc} r + 1 \geq 3 & \text{or} & r + 1 \leq -3 \\ r \geq 2 & \text{or} & r \leq -4 \end{array}$$

Solution Set: $(-\infty, -4] \cup [2, \infty)$

(c) (7 pts.) $|6 - 4t| < 10$

To satisfy the absolute value inequality, we must have

$$\begin{array}{l} -10 < 6 - 4t < 10 \\ -16 < -4t < 4 \quad \text{(Subtracting 6 from each part.)} \\ 4 > t > -1 \quad \text{(Dividing each part by } -4\text{.)} \end{array}$$

Solution Set: $(-1, 4)$

4. (24 pts.) Solve the following systems of equations by substitution or elimination.

(a) (12 pts.) $\begin{array}{l} \frac{3}{4}x + 3y = \frac{5}{2} \\ -4x + 9y = -5 \end{array}$

To get rid of the fractions, multiply the first equation by 4.
New System: $\begin{array}{l} 3x + 12y = 10 \\ -4x + 9y = -5 \end{array}$

Using elimination to eliminate x: $\begin{array}{r} 12x + 48y = 40 \\ \underline{-12x + 27y = -15} \\ 75y = 25 \\ y = \frac{1}{3} \end{array}$

Solving for x: $3x + 12(\frac{1}{3}) = 10 \quad \Rightarrow \quad 3x + 4 = 10 \quad \Rightarrow \quad 3x = 6$
$\Rightarrow \quad x = 2$

Solution Set: $(2, \frac{1}{3})$

(b) (12 pts.) $2x + 6y = 10$
$4x = -12y + 20$

Solving for x in the second equation, we have $x = -3y + 5$.

Using substitution in the first equation: $2(-3y + 5) + 6y = 10$
$-6y + 10 + 6y = 10$
$10 = 10$

Solution Set: $\{(x, y)|2x + 6y = 10\}$ (Infinite number of solutions.)

5. (20 pts.) Solve the following compound inequalities. Write your solutions in interval notation (as simplified as possible).

(a) (10 pts.) $-2x \geq -8$ or $x + 2 < 10$

Solving the inequalities separately:

- $-2x \geq -8 \quad \Rightarrow \quad x \leq 4$
- $x + 2 < 10 \quad \Rightarrow \quad x < 8$

The solutions to the compound inequality are solutions of the first inequality OR the second inequality. So, the solution set will be the union of the two individual solution sets.

Solution Set: $(-\infty, 8)$

(b) (10 pts.) $3m + 1 \geq -11$ and $m - 3 \leq -1$

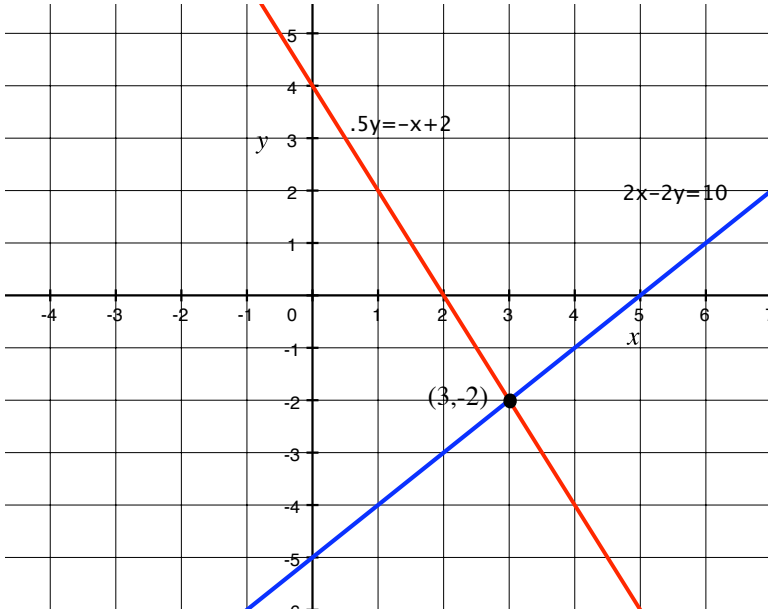
Solving the inequalities separately:

- $3m + 1 \geq -11 \quad \Rightarrow \quad 3m \geq -12 \quad \Rightarrow \quad m \geq -4$
- $m - 3 \leq -1 \quad \Rightarrow \quad m \leq 2$

The solutions to the compound inequality are solutions of the first inequality AND the second inequality. So, the solution set will be the intersection of the two individual solution sets.

Solution Set: $[-4, 2]$

6. (11 pts.) Solve the system $\begin{array}{l} 2x-2y=10 \\ \frac{1}{2}y=-x+2 \end{array}$ by graphing. (Be sure to check your solutions!)



The solution looks likes the point $(3, -2)$.

Checking the solution:

Equation 1: $2(3) - 2(-2) = 6 + 4 = 10$ ✓

Equation 2: $\frac{1}{2}(-2) = -3 + 2$ ✓

Solution Set: $\{(3, -2)\}$