

Math 99 Exam 4 Solutions

1. (15 pts.) $f(x) = 2x^2 + 8x + 5$

- (a) (5 pts.) The graph of f is a parabola. What is the vertex of the parabola?

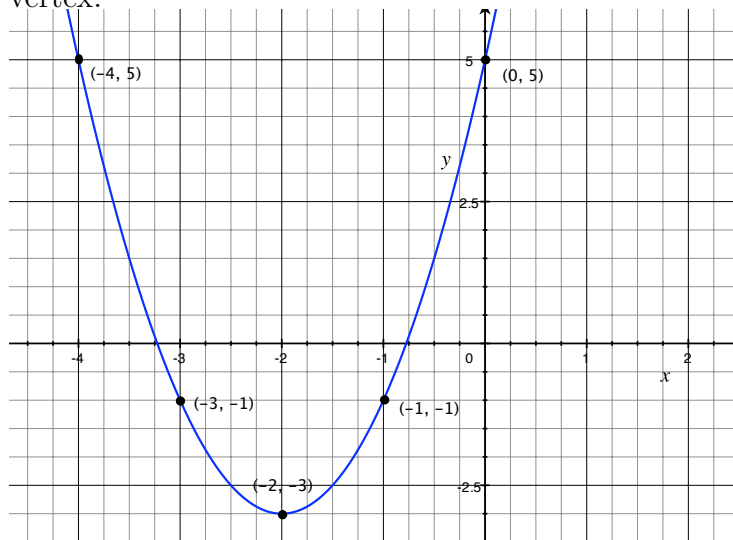
Vertex Formula:

$$x\text{-coordinate of the vertex: } x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$$

$$y\text{-coordinate of the vertex: } y = f(-2) = 2(-2)^2 + 8(-2) + 5 = -3$$

So the vertex is $(-2, -3)$.

- (b) (10 pts.) Draw the graph of f on the axis below. Include 2 points other than the vertex.



2. (20 pts.) Evaluate the following.

- (a) (5 pts.) $\log_5 125 = ?$

$$\text{Since } 5^3 = 125, \log_5 125 = 3.$$

- (b) (5 pts.) $\log_{11} 11 = ?$

$$\text{Since } 11^1 = 11, \log_{11} 11 = 1. \text{ (In fact, we have that } \log_a a = 1 \text{ for all } a > 0, a \neq 1.)$$

- (c) (5 pts.) $\log_4 1 = ?$

$$\text{Since } 4^0 = 1, \log_4 1 = 0. \text{ (In fact, we have that } \log_a 1 = 0 \text{ for all } a > 0, a \neq 1.)$$

(d) (5 pts.) $\ln 100 - \log 100 = ?$ (Give an approximation to four decimal places.)

$$\ln 100 - \log 100 \approx 2.6052 \quad \text{since } \ln 100 \approx 4.6052 \text{ and } \log 100 = 2.$$

3. (20 pts.) Fill in the following table by writing the exponential or logarithmic form for the corresponding equation.

	Exponential Form	Logarithmic Form
a	$2^4 = 16$	$\log_2 16 = 4$
b	$3^{-5} = \frac{1}{243}$	$\log_3 \frac{1}{243} = -5$
c	$7^3 = 343$	$\log_7 343 = 3$
d	$10^{-2} = \frac{1}{100}$	$\log \frac{1}{100} = -2$

4. (20 pts.) Solve the following equations for the variable x .

(a) (5 pts.) $4^x = \frac{1}{16}$

Since $\frac{1}{16} = 4^{-2}$, we can modify the right side of the original equation to get:

$$4^x = 4^{-2}$$

Both expressions have the same base. \Rightarrow The exponents are equal. $\Rightarrow x = -2$

(b) (5 pts.) $9^{6x} = 27^{x+1}$

Both 9 and 27 are powers of 3 ($9 = 3^2$, $27 = 3^3$), so $9^{6x} = 27^{x+1}$

$$\begin{aligned} \Rightarrow (3^2)^{6x} &= (3^3)^{x+1} \\ \Rightarrow 3^{12x} &= 3^{3x+3} \\ \Rightarrow 12x &= 3x + 3 \\ \Rightarrow x &= \frac{1}{3} \end{aligned}$$

(c) (5 pts.) $\log_8 x = 2$

Rewriting this equation in exponential form, we have $8^2 = x$ or $x = 64$.

(d) (5 pts.) $\log_x 216 = 3$

Rewriting this equation in exponential form, we have $x^3 = 216$
 $\Rightarrow x = 216^{1/3} = 6.$

5. (10 pts.) You fire a cannonball upward so that its distance (in feet) above the ground t seconds after firing is given by $h(t) = -16t^2 + 144t$. Find the maximum height it reaches and the number of seconds it takes to reach that height.

Note that the graph of the function h that describes the height of the cannonball is a downward-facing parabola. Thus, the maximum height of the cannonball occurs at the vertex of the parabola.

Using the vertex formula, we have that $t = \frac{-b}{2a} = \frac{-144}{2(-16)} = 4.5$ seconds.

So, the cannonball reaches its maximum height 4.5 seconds after it is fired and the maximum height is $h(4.5) = -16(4.5)^2 + 144(4.5) = 324$ feet.

6. (15 pts.) A survey of the squirrel population on the Shoreline campus is conducted and it is found that 1200 squirrels are present on campus this quarter.* It is projected that the population of squirrels on campus can be described by the following exponential function: $s(t) = 1200(3)^t$ where t is the number of years after the survey is conducted.

- (a) (8 pts.) How many squirrels can we expect to have on campus 8 years after the survey?

$$\begin{aligned}\text{Predicted number of squirrels after 8 years} &= s(8) = 1200(3)^8 \\ &= 1200(6561) \\ &= 7,873,200 \text{ squirrels}\end{aligned}$$

- (b) (7 pts.) When can we expect to have 97200 squirrels on campus?

We are trying to find a time t for which $s(t) = 97200$.

$$\begin{aligned}\text{Setting } s(t) \text{ equal to } 97200 &\Rightarrow 97200 = 1200(3)^t \\ 81 &= 3^t \quad (\text{Dividing both sides by } 1200) \\ 3^4 &= 3^t \quad (\text{Since } 3^4 = 81) \\ \Rightarrow t &= 4\end{aligned}$$

The squirrel population is expected to be 97200 after 4 years.

*Note: This survey and population estimate is completely fictional and no squirrels were harmed in this fictional survey.