

Math 99 Exam 3 Solutions

1. (20 pts.) Simplify the following:

(a) (5 pts.) i^{177}

$$i^{177} = i^{176} \cdot i = (i^2)^{88} \cdot i = (-1)^{88} \cdot i = 1 \cdot i = i$$

(b) (5 pts.) $(-2 + 4i)(3 - 2i)$

$$(-2 + 4i)(3 - 2i) = -6 + 4i + 12i - 8i^2 = -6 + 16i + 8 = 2 + 16i$$

(c) (5 pts.) $(-2 + 4i) + (3 - 2i)$

$$(-2 + 4i) + (3 - 2i) = -2 + 3 + (4 - 2)i = 1 + 2i$$

(d) (5 pts.) $\frac{5}{2-i}$

$$\frac{5}{2-i} = \frac{5}{2-i} \cdot \frac{2+i}{2+i} = \frac{5(2+i)}{4-i^2} = \frac{5(2+i)}{4-(-1)} = \frac{5(2+i)}{5} = 2 + i$$

2. (25 pts.) For the following problems, **simplify** your answers as much as possible.

(a) (8 pts.) Solve for p : $-3(p+4)^2 = 27$

Dividing both sides by -3 : $(p+4)^2 = -9$
 Using the square root property: $p+4 = \sqrt{-9} = 3i$ OR $p+4 = -\sqrt{-9} = -3i$
 Solving for x in both equations: $p = -4 + 3i$ OR $p = -4 + 3i$
 $(p = -4 \pm 3i)$

(b) (9 pts.) Complete the square to solve for x : $4x^2 + 8 = 20x$

Standard Form: $4x^2 - 20x + 8 = 0$

Dividing both sides of the equation by 3: $x^2 - 5x + 2 = 0$
 Subtracting 3 from both sides: $x^2 - 5x = -2$
 Completing the square: $x^2 - 5x + \frac{25}{4} = -2 + \frac{25}{4}$
 $(x - \frac{5}{2})^2 = \frac{17}{4}$
 Using the square root property: $x - \frac{5}{2} = \frac{\sqrt{17}}{2}$ OR $x - \frac{5}{2} = -\frac{\sqrt{17}}{2}$
 $x = \frac{5+\sqrt{17}}{2}$ OR $r = \frac{5-\sqrt{17}}{2}$
 $r = \frac{5 \pm \sqrt{17}}{2}$

(c) (8 pts.) Use the quadratic formula to solve for r : $-3r^2 + 8r - 1 = 0$

$$a = -3, b = 8, c = -1$$

$$\begin{aligned} \text{Solutions are } r &= \frac{-8 \pm \sqrt{8^2 - 4(-3)(-1)}}{2(-3)} = \frac{-8 \pm \sqrt{64 - 12}}{-6} = \frac{-8 \pm \sqrt{52}}{-6} = \frac{-8 \pm 2\sqrt{13}}{-6} \\ &= \frac{4 \pm \sqrt{13}}{3} \end{aligned}$$

3. (20 pts.) Walking along the edge of a cliff 64 feet high, you notice a cannon loaded with a cannonball. Feeling rebellious and seeing that there is no one in the area below, you decide to fire the cannonball. Knowing some physics, you find that the height (in feet) of the cannonball above the ground at a given time t (in seconds) is modeled by the function $s(t) = -16t^2 + 32t + 64$.

- (a) (10 pts.) How high above the ground is the cannonball after 1 second?

Since $s(t)$ = height of cannonball at time t , $s(1)$ = height of cannonball at 1 second.

$$s(1) = -16(1)^2 + 32(1) + 64 = -16 + 32 + 64 = 80 \text{ feet}$$

- (b) (10 pts.) At what time(s) is the cannonball 64 feet above the ground?

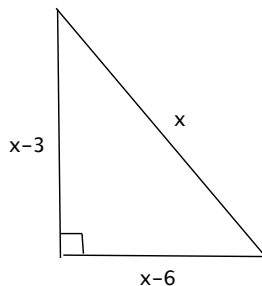
We are looking for time(s) t such that $s(t) = 64$.

$$\begin{aligned} 64 &= -16t^2 + 32t + 64 \quad \Rightarrow \quad 0 = -16t^2 + 32t \\ &\qquad\qquad\qquad 0 = -16t(t - 2) \end{aligned}$$

$$\text{So, either } -16t = 0 \text{ or } t - 2 = 0 \quad \Rightarrow \quad t = 0 \text{ or } t = 2$$

The cannonball is 64 feet above the ground at $t = 0$ seconds and $t = 2$ seconds.

4. (15 pts.) Find the lengths of the sides of the following right triangle.



$$\begin{aligned} \text{Using the Pythagorean Theorem: } (x - 6)^2 + (x - 3)^2 &= x^2 \\ x^2 - 12x + 36 + x^2 - 6x + 9 &= x^2 \\ x^2 - 18x + 45 &= 0 \\ (x - 15)(x - 3) &= 0 \end{aligned}$$

So either $x = 15$ or $x = 3$. Note: The choice of $x = 3$ leads to the following lengths of the sides of the triangle: 3, 0, -3 \Leftarrow Impossible.

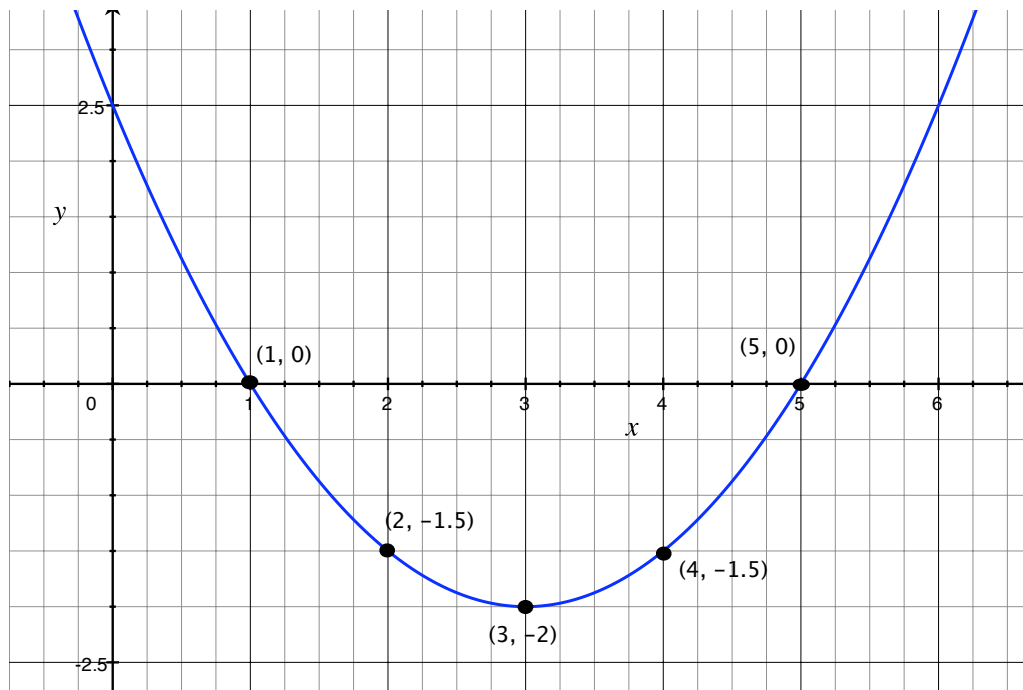
With the choice of $x = 15$, we have that the lengths of the sides are 15, 12, and 9.

5. (20 pts) $f(x) = \frac{1}{2}(x - 3)^2 - 2$

(a) (5 pts.) What is the vertex of f ?

The vertex form of a parabola is $a(x - h)^2 + k$ where the vertex is the point (h, k) . Since $f(x)$ is already in vertex form, we can see that the vertex is $(3, -2)$.

(b) (10 pts.) Sketch a graph of f below. Include 2 points on the graph other than the vertex.



(c) (5 pts.) What is the range of f ?

Range of $f = [-2, \infty)$