Math 99 Exam 3 Solutions

1. (20 pts.) Simplify the following. Recall that $i^2 = -1$.

   (a) (5 pts.) i^{143}

   $i^{143} = i^{142} \cdot i = (i^2)^{71} \cdot i = (-1)^{71} \cdot i = -1 \cdot i = -i$

   (b) (5 pts.) $(2+3i)(-1-5i)$

   $(2+3i)(-1-5i) = -2-10i-3i-15i^2 = -2-13i-15(-1) = -2-13i+15 = 13-13i$

   (c) (5 pts.) $(2+3i)+(-1-5i)$

   $(2+3i)+(-1-5i) = 2-1+(3-5)i = 1-2i$

   (d) (5 pts.) $\frac{4}{1+i}$

   $\frac{4}{1+i} = \frac{4}{1+i} \cdot \frac{1-i}{1-i} = \frac{4-4i}{1-i^2} = \frac{4-4i}{1-(-1)} = \frac{4-4i}{2} = 2-2i$

2. (25 pts.) For the following problems, **simplify** your answers as much as possible.

   (a) (8 pts.) Solve for x: $-2(x-3)^2 = 8$

   Dividing both sides by -2: $(x-3)^2 = -4$
   Using the square root property: $x-3 = \sqrt{-4} = 2i$ OR $x-3 = -\sqrt{-4} = -2i$
   Solving for x in both equations: $x = 3+2i$ OR $x = 3-2i$
   $(x = 3 \pm 2i)$

   (b) (9 pts.) Complete the square to solve for r: $3r^2 + 15r + 9 = 0$

   Dividing both sides of the equation by 3: $r^2 + 5r + 3 = 0$
   Subtracting 3 from both sides: $r^2 + 5r = -3$
   Completing the square: $r^2 + 5r + \frac{25}{4} = -3 + \frac{25}{4}$
   $(r + \frac{5}{2})^2 = \frac{13}{4}$
   Using the square root property: $r + \frac{5}{2} = \frac{\sqrt{13}}{2}$ OR $r + \frac{5}{2} = -\frac{\sqrt{13}}{4}$
   $r = \frac{-5+\sqrt{13}}{2}$ OR $r = \frac{-5-\sqrt{13}}{2}$
   $r = \frac{-5\pm\sqrt{13}}{2}$

   (c) (8 pts.) Use the quadratic formula to solve for t: $-2t^2 + 6t = 1$

   Standard Form: $-2t^2 + 6t - 1 = 0$ $(a = -2,\ b = 6,\ c = -1)$

Solutions are $t = \frac{-6 \pm \sqrt{6^2 - 4(-2)(-1)}}{2(-2)} = \frac{-6 \pm \sqrt{36-8}}{-4} = \frac{-6 \pm \sqrt{28}}{-4} = \frac{-6 \pm 2\sqrt{7}}{4}$

$$= \frac{3 \pm \sqrt{7}}{2}$$

3. (20 pts.) Being a natural-born rebel, you decide to toss a coin from the top of a building 48 feet high. Since you are a rebel who knows physics, you find that the height (in feet) of the coin above the ground at a given time t (in seconds) is modeled by the function $s(t) = -16t^2 + 8t + 48$.

   (a) (10 pts.) How high above the ground is the coin after 1 second?

   Since $s(t)$=height of coin at time t, $s(1)$=height of coin at 1 second.

   $s(1) = -16(t)^2 + 8(1) + 48 = -16 + 8 + 48 = 40$ feet

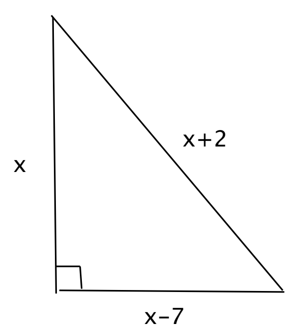
   (b) (10 pts.) At what time(s) is the coin 48 feet above the ground?

   We are looking for time(s) t such that $s(t) = 48$.

   $$48 = -16t^2 + 8t + 48 \quad \Rightarrow \quad 0 = -16t^2 + 8t$$
   $$0 = -8t(2t-1)$$
   $$\text{So, either } -8t = 0 \text{ or } 2t - 1 = 0 \quad \Rightarrow \quad t = 0 \text{ or } t = \tfrac{1}{2}$$

   The coin is 48 feet above the ground at $t = 0$ seconds and $t = \frac{1}{2}$ seconds.

4. (15 pts.) Find the lengths of the sides of the following right triangle.



Using the Pythagorean Theorem: $x^2 + (x-7)^2 = (x+2)^2$

$$x^2 + x^2 - 14x + 49 = x^2 + 4x + 4$$
$$x^2 - 18x + 45 = 0$$
$$(x-15)(x-3) = 0$$

So either $x = 15$ or $x = 3$. Note: The choice of $x = 3$ leads to the following lengths of the sides of the triangle: 3, -4, 5 $\Leftarrow$ Impossible.

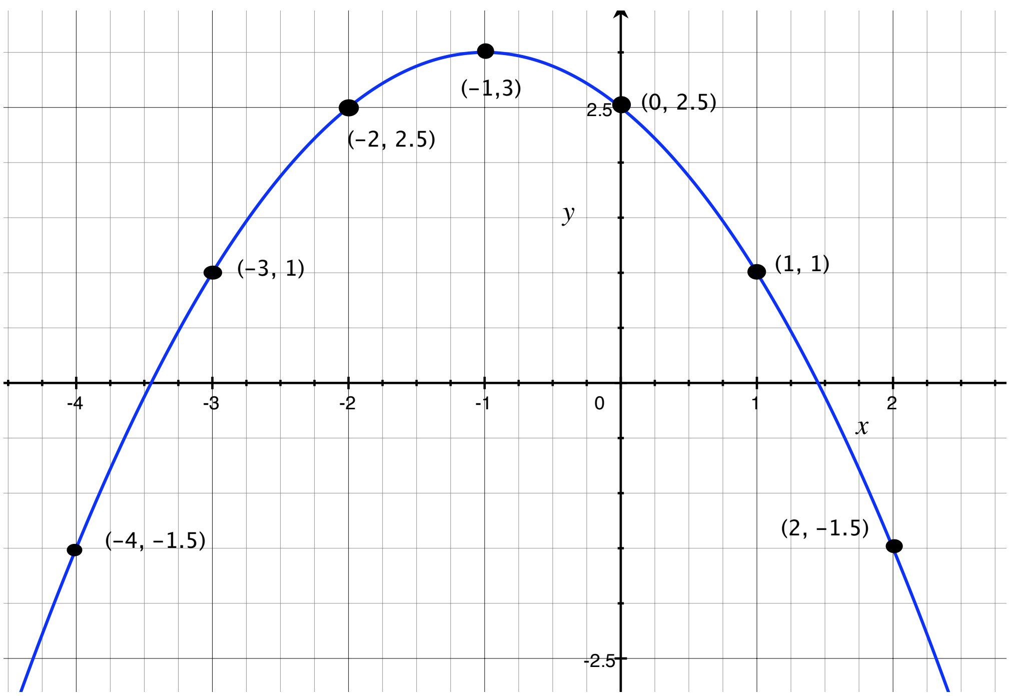
With the choice of $x = 15$, we have that the lengths of the sides are 15, 8, and 17.

5. (20 pts) $f(x) = -\frac{1}{2}(x+1)^2 + 3$

(a) (5 pts.) What is the vertex of f?

The vertex form of a parabola is $a(x-h)^2 + k$ where the vertex is the point (h, k). Since $f(x)$ is already in vertex form, we can see that the vertex is $(-1, 3)$.

(b) (10 pts.) Sketch a graph of f below. Include 2 points on the graph other than the vertex.



(c) (5 pts.) What is the range of f?

Range of $f = (-\infty, 3]$