Math 99
Exam 2

1. (15 pts.) Solve the following inequalities. Give your answer in both interval and graph form.

   - $|2x+3| \leq 15$

     $|2x+3| \leq 15 \quad \Rightarrow \quad -15 \leq 2x+3 \leq 15 \quad \Rightarrow \quad -18 \leq 2x \leq 12 \quad \Rightarrow \quad -9 \leq x \leq 6$

     Interval form of solution: $[-9, 6]$
     Graph:
     ←- - - - - - - - - - - - -[————————————————————]- - - - - - - - - - - - - - -→
     -9 6

   - $|2x+3| > 15$

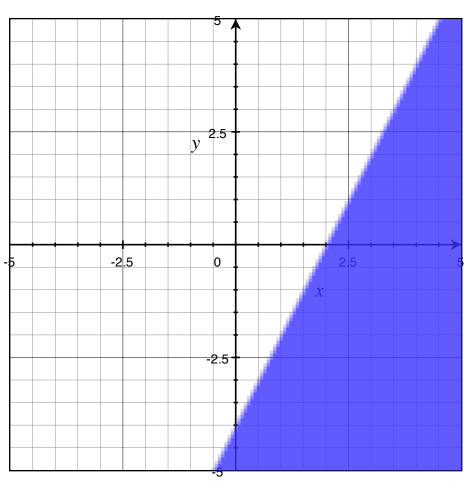
     $|2x+3| > 15 \quad \Rightarrow \quad 2x+3 > 15$ OR $2x+3 < -15$

     1) If $2x+3 > 15 \quad \Rightarrow \quad 2x > 12 \quad \Rightarrow x > 6$.

     2) If $2x+3 < -15 \quad \Rightarrow \quad 2x < -18 \quad \Rightarrow x < -9$

     Interval form of solution: $(-\infty, -9)$U$(6, \infty)$
     Graph:
     ←————————————)- -(——————————————→
     -9 6

2. (10 pts.) Graph the solutions to the inequality $2x - y > 4$.



3. (25 pts.) Simplify the following expressions (Assume all variables are positive):

(a) (6 pts.) $\left(\frac{x^3y^{-6}}{x^{-1}y^2}\right)^{-\frac{3}{4}}$ (Give answer with positive exponents only.)

$$\left(\frac{x^3y^{-6}}{x^{-1}y^2}\right)^{-\frac{3}{4}} = \left(\frac{x^3x^1}{y^6y^2}\right)^{-\frac{3}{4}} = \left(\frac{x^4}{y^8}\right)^{-\frac{3}{4}} = \left(\frac{y^8}{x^4}\right)^{\frac{3}{4}} = \frac{y^{8(\frac{3}{4})}}{x^{4(\frac{3}{4})}} = \frac{y^6}{x^3}$$

(b) (6 pts.) $-\sqrt{64a^4b^8}$

$-\sqrt{64a^4b^8} = -8a^2b^4$ since $\sqrt{64} = 8$, $\sqrt{a^4} = a^2$, $\sqrt{b^8} = b^4$

(c) (7 pts.) $3\sqrt{27x^3} + 2x\sqrt{75x}$

$$\begin{aligned} 3\sqrt{27x^3} + 2x\sqrt{75x} = 3\sqrt{9\cdot 3x^2\cdot x} + 2x\sqrt{25\cdot 3x} &= 3\cdot 3x\sqrt{3x} + 2x\cdot 5\sqrt{3x} \\ &= 9x\sqrt{3x} + 10x\sqrt{3x} \\ &= 19x\sqrt{3x} \end{aligned}$$

(d) (6 pts.) $\frac{\sqrt{18}}{3-\sqrt{2}}$ (Rationalize the denominator and simplify.)

$$\begin{aligned} \frac{\sqrt{18}}{3-\sqrt{2}} = \frac{\sqrt{18}}{3-\sqrt{2}}\cdot\frac{3+\sqrt{2}}{3+\sqrt{2}} &= \frac{\sqrt{18}(3+\sqrt{2})}{9-2} \text{ since } (3+\sqrt{2})(3-\sqrt{2}) = 3^2-(\sqrt{2})^2 = 9-2 \\ &= \frac{3\sqrt{18}+\sqrt{36}}{7} = \frac{9\sqrt{2}+6}{7} \end{aligned}$$

4. (20 pts.) Let $f(x) = x^2 - 3x + 2$

(a) (5 pts.) What is the domain of f?

All real numbers x $(-\infty, \infty)$

(b) (5 pts.) What is $f(3)$?
$f(3) = 3^2 - 3(3) + 2 = 9 - 9 + 2 = 2$

(c) (10 pts.) What is $f(x+3)$? Simplify your answer as much as possible.

$f(x+3) = (x+3)^2 - 3(x+3) + 2 = (x^2 + 6x + 9) + (-3x - 9) + 2 = x^2 + 3x + 2$

5. (20 pts.) Solve the following equations:

(a) (10 pts.) $\sqrt[3]{3x-1} - 2 = 0$

$\sqrt[3]{3x-1} - 2 = 0 \quad \Rightarrow \quad \sqrt[3]{3x-1} = 2$

Cubing both sides: $(\sqrt[3]{3x-1})^3 = 2^3 \quad \Rightarrow \quad 3x - 1 = 8 \quad \Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$

Checking the solution: $\sqrt[3]{3(3)-1} - 2 = \sqrt[3]{8} - 2 = 2 - 2 = 0$✓

(b) (10 pts.) $\sqrt{17-8x} = 3 - x$

Squaring both sides: $(\sqrt{17-8x})^2 = (3-x)^2$

$$\begin{aligned} 17 - 8x &= 9 - 6x + x^2 = x^2 - 6x + 9 \\ 0 &= x^2 + 2x - 8 \\ 0 &= (x+4)(x-2) \end{aligned}$$

So, $x = -4$ and $x = 2$ are possible solutions to the equation.

Checking the solutions:
$\underline{x = -4}$: $\sqrt{17-8(-4)} = \sqrt{17+32} = \sqrt{49} = 7 = 3 - (-4)$✓

$\underline{x = 2}$: $\sqrt{17-8(2)} = \sqrt{17-16} = \sqrt{1} = 1 = 3 - 2$✓

The equation has two solutions, $x = -4$ and $x = 2$.

6. (10 pts.) Find the distance between the points $(-3, 2)$ and $(1, -2)$. Simplify your answer.

$d = \sqrt{(-3-1)^2 + (2-(-2))^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$