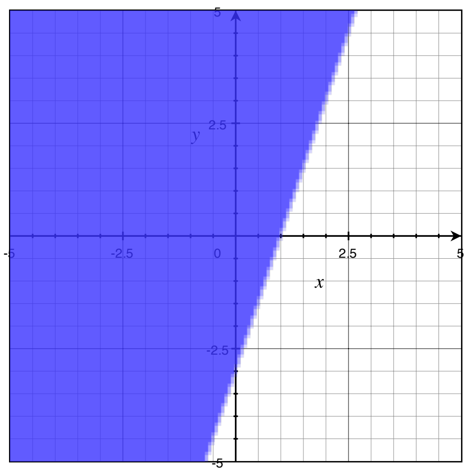
Math 99
Exam 2

1. (10 pts.) Graph the solutions to the inequality $3x - y < 3$.



2. (20 pts.) Let $f(x) = x^2 + 2x - 1$

   (a) (5 pts.) What is the domain of f?

   All real numbers x $\quad (-\infty, \infty)$

   (b) (5 pts.) What is $f(2)$?

   $f(2) = 2^2 + 2(2) - 1 = 7$

   (c) (10 pts.) What is $f(x + 2)$? Simplify your answer as much as possible.

   $f(x+2) = (x+2)^2 + 2(x+2) - 1 = (x^2 + 4x + 4) + (2x + 4) - 1 = x^2 + 6x + 7$

3. (40 pts.) Simplify the following expressions (Assume all variables are positive):
   Note: Often times, there is more than one way to simplify each expression, but the final simplified answer will still be the same.*

   (a) (6 pts.) $\sqrt[6]{y^5} \cdot \sqrt[3]{y^2}$ (Give answer in exponential form.)

   $\sqrt[6]{y^5} \cdot \sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{5/6+4/6} = y^{9/6} = y^{3/2}$

   (b) (7 pts.) $\left(\dfrac{a^{-2}b^3}{a^4b^{-3}}\right)^{-\frac{2}{3}}$ (Give answer with positive exponents only.)

$$\left(\frac{a^{-2}b^3}{a^4b^{-3}}\right)^{-\frac{2}{3}}=\left(\frac{b^3b^3}{a^4a^2}\right)^{-\frac{2}{3}}=\left(\frac{b^{3+3}}{a^{4+2}}\right)^{-\frac{2}{3}}=\left(\frac{b^6}{a^6}\right)^{-\frac{2}{3}}=\left(\frac{a^6}{b^6}\right)^{\frac{2}{3}}=\frac{a^{6(\frac{2}{3})}}{b^{6(\frac{2}{3})}}=\frac{a^4}{b^4}$$

(c) (6 pts.) $-\sqrt{100x^6y^4}$

$-\sqrt{100x^6y^4} = -10x^3y^2$ since $\sqrt{100} = 10$, $\sqrt{x^6} = x^3$, $\sqrt{y^4} = y^2$

(d) (7 pts.) $2\sqrt{18a^3} + 5a\sqrt{72a}$

$$\begin{aligned} 2\sqrt{18a^3} + 5a\sqrt{72a} = 2\sqrt{9 \cdot 2a^2 \cdot a} + 5a\sqrt{36 \cdot 2a} &= 2 \cdot 3a\sqrt{2a} + 5a \cdot 6\sqrt{2a} \\ &= 6a\sqrt{2a} + 30a\sqrt{2a} \\ &= 36a\sqrt{2a} \end{aligned}$$

(e) (7 pts.) $\frac{\sqrt{8}}{2-\sqrt{2}}$ (Rationalize the denominator and simplify.)

$$\begin{aligned} \frac{\sqrt{8}}{2-\sqrt{2}} = \frac{\sqrt{8}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}} &= \frac{\sqrt{8}(2+\sqrt{2})}{4-2} \text{ since } (2+\sqrt{2})(2-\sqrt{2}) = 2^2-(\sqrt{2})^2 = 4-2 \\ &= \frac{2\sqrt{8}+\sqrt{16}}{2} \\ &= \frac{4\sqrt{8}+4}{2} = 2\sqrt{2}+2 \end{aligned}$$

(f) (7 pts.) $\sqrt{\frac{9}{x^6}}+\sqrt{\frac{1}{25}}$ (Write the answer as one fraction.)

$$\sqrt{\frac{9}{x^6}}+\sqrt{\frac{1}{25}} = \frac{\sqrt{9}}{\sqrt{x^6}}+\frac{\sqrt{1}}{\sqrt{25}} = \frac{3}{x^3}+\frac{1}{5} = \frac{15}{5x^3}+\frac{x^3}{5x^3} = \frac{15+x^3}{5x^3}$$

4. (20 pts.) Solve the following equations:

(a) (10 pts.) $\sqrt[3]{2x+1} - 3 = 0$

$\sqrt[3]{2x+1} - 3 = 0 \quad \Rightarrow \quad \sqrt[3]{2x+1} = 3$

Cubing both sides: $2x + 1 = 27 \quad \Rightarrow \quad 2x = 26 \quad \Rightarrow \quad x = 13$

Checking the solution: $\sqrt[3]{2 \cdot 13 + 1} - 3 = \sqrt[3]{27} - 3 = 3 - 3 = 0$✓

(b) (10 pts.) $\sqrt{16 - 5x} = x - 2$

$$\begin{aligned} \text{Squaring both sides: } (\sqrt{16-5x})^2 &= (x-2)^2 \\ 16 - 5x &= x^2 - 4x + 4 \\ 0 &= x^2 + x - 12 \\ 0 &= (x+4)(x-3) \end{aligned}$$

So, $x = -4$ and $x = 3$ are possible solutions to the equation.

Checking the solutions:
$\underline{x=-4}$: $\sqrt{16-5(-4)} = \sqrt{16+20} = \sqrt{36} = 6 \neq -4-2 \;\Rightarrow\; x=-4$ is not a solution.

$\underline{x=3}$: $\sqrt{16-5(3)} = \sqrt{16-15} = \sqrt{1} = 1 = 3-2$✓

The only solution to the equation is $x = 3$.

5. (10 pts.) Find the distance between the points $(1,-3)$ and $(-4,2)$. Simplify your answer.

$$d = \sqrt{(1-(-4))^2 + (-3-2)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$