Math 99
Exam 1 Solutions

1. (30 pts.) What are the solutions to the following systems of equations?

   (a) (10 pts.) $2x + 10y = 3$
   $x = 1 - 5y$

   Using substitution, we have $2(1-5y)+10y = 3 \Rightarrow 2-10y+10y = 3 \Rightarrow 2 = 3$.
   False statement $\Rightarrow$ No solutions $\emptyset$

   (b) (10 pts.) $4x - 3y = -7$
   $3x + 2y = 16$

   Eliminating y by multiplying the top equation by 2 and the bottom by 3:
   $8x - 6y = -14$
   $9x + 6y = 48$
   $17x = 34 \quad \Rightarrow \quad x = 2$

   Plugging $x = 2$ back in: $4(2) - 3y = -7 \Rightarrow y = 5$. Solution: $(2, 5)$

   (c) (10 pts.) $6x - 2y = 24$
   $y = 3x - 12$

   Using substitution, we have $6x - 2(3x - 12) = 24 \Rightarrow 6x - 6x + 24 = 24 \Rightarrow 24 = 24$.
   This is a statement that is true for all x, so we must have that the two equations represent the same line $\Rightarrow$ Infinite number of solutions to the system.

2. (20 pts.) A blue car and a red car leave from the same point, traveling in opposite directions at constant speeds. The blue car drives East and the red car drives West **15 mph faster** than the blue car. If the cars are **460 miles apart** after **4 hours**, find the speed of each car by writing a linear system that models the situation.

   Red Car x mph Blue Car y mph
   ←- -Leaving point- - - - - - - - - - - - - - - - - - →

   |460 miles after 4 hours|

   System: $x = y + 15$
   $4x + 4y = 460 \quad \Rightarrow \quad y = 50 \Rightarrow \quad x = 65$
   The red car drives at 65 mph and the blue car drives at 50 mph.

3. (5 pts.) Is $(1, 2, -3)$ a solution to the system $4x - y + 2z = 3$?
$$x - y - z = 2$$
$$2x + 3y + z = 5$$

No, because the values $x = 1$, $y = 2$, $z = -3$ do not satisfy the first equation. $4(1) - 2 + 2(-3) \neq 3$

4. (20 pts.) Solve the following compound inequalities and give the solutions in both interval and graph forms.

(a) (10 pts.) $3x + 2 \leq 11$ and $-2x \leq 4$

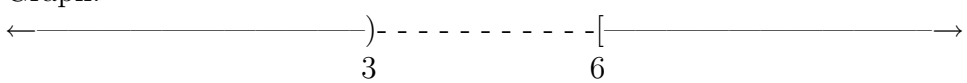
Solving the first inequality: $3x + 2 \leq 11 \Rightarrow 3x \leq 9 \Rightarrow x \leq 3$
Solving the second inequality: $-2x \leq 4 \Rightarrow x \geq -2$

Graph:


Interval: $[-2, 3]$

(b) (10 pts.) $x - 2 < 1$ or $5 + \frac{1}{2}x \geq 8$

Solving the first inequality: $x - 2 < 1 \Rightarrow x < 3$
Solving the second inequality: $5 + \frac{1}{2}x \geq 8 \Rightarrow \frac{1}{2}x \geq 3 \Rightarrow x \geq 6$

Graph:


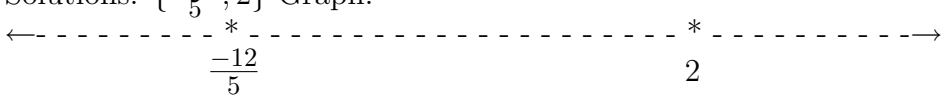
Interval: $(-\infty, 3) \cup [6, \infty)$

5. (25 pts.) Solve the following absolute value equalities and inequalities and give the solutions in both interval and graph forms.

(a) (8 pts.) $|5x + 1| = 11$

Two possibilities:

- $5x + 1 = 11 \Rightarrow 5x = 10 \Rightarrow x = 2$
- $5x + 1 = -11 \Rightarrow 5x = -12 \Rightarrow x = \frac{-12}{5}$

Solutions: $\{\frac{-12}{5}, 2\}$ Graph:


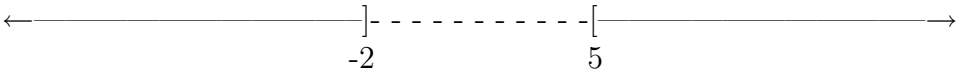
(b) (8 pts.) $|2x - 3| \geq 7$

Two possibilities:

- $2x - 3 \geq 7 \quad \Rightarrow \quad 2x \geq 10 \quad \Rightarrow x \geq 5$
- $2x - 3 \leq -7 \quad \Rightarrow \quad 2x \leq -4 \quad \Rightarrow x \leq -2$

Solutions: $(-\infty, -2]\text{U}[5, \infty)$

Graph:



(c) (9 pts.) $| - \frac{1}{2}x + 3| < 4$

We must have $-4 < -\frac{1}{2}x + 3 < 4 \;\Rightarrow\; -7 < -\frac{1}{2}x < 1 \;\Rightarrow\; 14 > x > -2$

Solutions: $(-2, 14)$

Graph: