

Math 98 Worksheet #8 Solutions

1. $f(x) = -\frac{1}{3}(x + 4)^2 + 1$

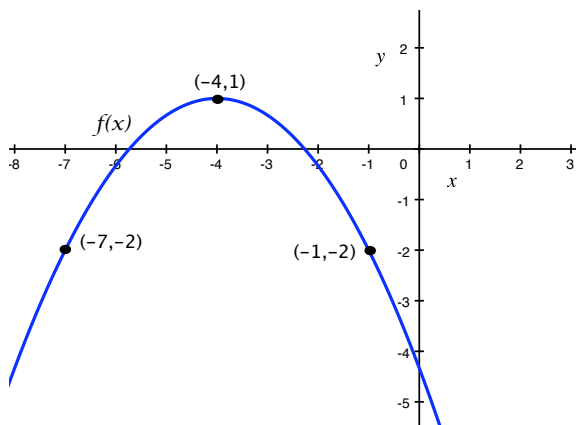
(a) What is the vertex of the graph of f ?

Vertex: $(-4, 1)$

(b) Does f open up or down? Is it narrower or wider than x^2 ?

Since $(x + 4)^2$ has a coefficient of $-\frac{1}{3}$, the parabola opens down and is narrower than x^2 .

(c) Sketch a graph of f including two points other than the vertex.



2. $g(x) = x^2 - 12x + 40$

(a) Write $g(x) = x^2 - 12x + 40$ in vertex form by completing the square.

Note that $(x - 6)^2 = x^2 - 12x + 36$.

$$\begin{aligned} g(x) &= x^2 - 12x + 40 = x^2 - 12x + 36 - 36 + 40 \\ &= (x - 6)^2 - 36 + 40 \\ &= (x - 6)^2 - 4 \quad \leftarrow \text{Vertex Form with } a = 1, h = 6, k = -4 \end{aligned}$$

(b) What is the vertex of the graph of g ?

The vertex of g is $(6, -4)$.

(c) What is the domain and range of g ?

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$ (The graph of g opens up.)

3. Use the vertex formula to find the vertex of the following quadratic functions. Does the parabola open up or down? Is it narrower or wider than x^2 ?

(a) $h(x) = 3x^2 + 6x - 7$

x -coordinate of vertex: $x = -\frac{6}{2(3)} = -\frac{6}{6} = -1$

y -coordinate of vertex: $y = h(-1) = 3(-1)^2 + 6(-1) - 7$
 $= 3 - 6 - 7$
 $= -10$

So, the vertex is $(-1, -10)$.

Since the coefficient of the squared term is 3, the parabola opens up and is narrower than x^2 .

(b) $F(x) = -2x^2 + 12x + 2$

x -coordinate of vertex: $x = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3$

y -coordinate of vertex: $y = F(3) = -2(3)^2 + 12(3) + 2$
 $= -18 + 36 + 2$
 $= 20$

So, the vertex is $(3, 20)$.

Since the coefficient of the squared term is -2, the parabola opens down and is narrower than x^2 .

(c) $G(x) = x^2 + 5x$

x -coordinate of vertex: $x = -\frac{5}{2(1)} = -\frac{5}{2}$

y -coordinate of vertex: $y = G(-\frac{5}{2}) = (-\frac{5}{2})^2 + 5(-\frac{5}{2})$
 $= \frac{25}{4} - \frac{25}{2}$
 $= \frac{25}{4} - \frac{50}{4}$
 $= -\frac{25}{4} = -6.25$

So, the vertex is $(-\frac{5}{2}, -\frac{25}{4})$ or $(-2.5, -6.25)$.

Since the coefficient of the squared term is 1, the parabola opens up and is the same shape as x^2 .