

Math 98 Worksheet #6 Solutions

1. Solve the following equations.

(a) $6 - \frac{2}{x} = \frac{4}{x^2}$

Multiplying both sides by x^2 : $x^2(6 - \frac{2}{x}) = x^2(\frac{4}{x^2})$
 $6x^2 - x^2(\frac{2}{x}) = 4$
 $6x^2 - 2x = 4$
 $6x^2 - 2x - 4 = 0$
 $3x^2 - x - 2 = 0$ (Dividing both sides by 2.)
 $(3x + 2)(x - 1) = 0$

So, $3x = -2$ or $x = 1 \Rightarrow x = -\frac{2}{3}$ or $x = 1$.

(Note: You can also use the quadratic formula on $3x^2 - x - 2 = 0$ to get the same solutions.)

(b) $\frac{6}{y} - 2 = \frac{y}{y+1}$

Multiplying both sides by $y(y+1)$: $y(y+1)(\frac{6}{y} - 2) = y(y+1)(\frac{y}{y+1})$
 $y(y+1)\frac{6}{y} - 2y(y+1) = y^2$
 $6(y+1) - 2y(y+1) = y^2$
 $6y + 6 - 2y^2 - 2y = y^2$
 $-3y^2 + 4y + 6 = 0$

Using the quadratic formula: $y = \frac{-4 \pm \sqrt{4^2 - 4(-3)(6)}}{2(-3)}$
 $= \frac{-4 \pm \sqrt{16 + 72}}{-6}$
 $= \frac{-4 \pm \sqrt{88}}{-6}$
 $= \frac{-4 \pm 2\sqrt{22}}{-6}$ ($\sqrt{88} = \sqrt{4 \cdot 22} = 2\sqrt{22}$)
 $= \frac{2 \pm \sqrt{22}}{3}$ (Dividing the numerator and denominator by -2.)

(c) $m^4 - 17m^2 = -72$

Putting all terms on one side: $m^4 - 17m^2 + 72 = 0$

We can factor this into the following: $(m^2 - 8)(m^2 - 9) = 0$
 $\Rightarrow m^2 = 8$ or $m^2 = 9$
 $\Rightarrow m = \pm\sqrt{8}$ or $m = \pm\sqrt{9}$
 $\Rightarrow m = \pm 2\sqrt{2}$ or $m = \pm 3$

So, we have four solutions: $m = 2\sqrt{2}, -2\sqrt{2}, 3, -3$

2. Solve the following equations for the indicated variable.

(a) $F = \frac{m}{r^2}$ for r

Multiplying both sides by r^2 (to get it out of the denominator): $r^2F = m$

Dividing both sides by F : $r^2 = \frac{m}{F}$
 $r = \pm\sqrt{\frac{m}{F}} = \pm\frac{\sqrt{mF}}{F}$

(b) $k = 2s\sqrt{d}$ for d

Dividing both sides by $2s$: $\frac{k}{2s} = \sqrt{d}$

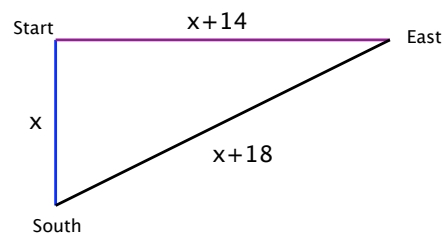
Squaring both sides: $\frac{k^2}{4s^2} = d$

3. Two cars left an intersection at the same time. One traveled south and the other traveled east. After a certain amount of time, the eastbound car has traveled 14 miles further than the southbound car. How far has each car traveled if the distance between them was 4 miles more than the distance the east bound car has traveled?

Here is a couple of ways to set up the equations:

- Let x = Distance traveled by the southbound car
 $\Rightarrow x + 14$ = Distance traveled by the eastbound car
 $x + 18$ = Distance between the cars

We have the following diagram:



Equation: $x^2 + (x + 14)^2 = (x + 18)^2$
 $x^2 + x^2 + 28x + 196 = x^2 + 36x + 324$
 $x^2 - 8x - 128 = 0$

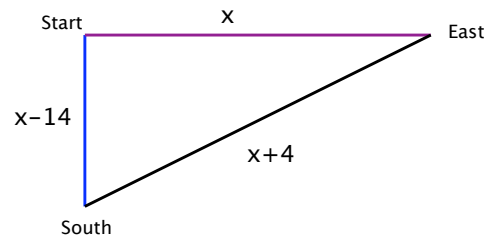
You can factor this into $(x - 16)(x + 8) = 0$ and get the solutions $x = 16$ and $x = -8$ or you can use the quadratic formula to obtain the same solutions.

Only the solution of $x = 16$ makes sense in our situation since distances should be positive. So, the southbound car travels 16 miles, the eastbound car travels 30 miles, and the distance between the cars is 34 miles.

Check: $16^2 + 30^2 = 34^2$ ✓

- Let $x =$ Distance traveled by the eastbound car
 $\Rightarrow x - 14 =$ Distance traveled by the southbound car
 $x + 4 =$ Distance between the cars

We have the following diagram:



Equation: $(x - 14)^2 + x^2 = (x + 4)^2$
 $x^2 - 28x + 196 + x^2 = x^2 + 8x + 16$
 $x^2 - 36x + 180 = 0$

You can factor this into $(x - 6)(x - 30) = 0$ and get the solutions $x = 6$ and $x = 30$ or you can use the quadratic formula to obtain the same solutions.

Only the solution of $x = 30$ makes sense in our situation since distances should be positive and $x = 6$, then the southbound car will have traveled -8 miles, which does not make sense in our situation. So, the southbound car travels 16 miles, the eastbound car travels 30 miles, and the distance between the cars is 34 miles.

Check: $16^2 + 30^2 = 34^2$ ✓