

## Math 98 Worksheet #3 Solutions

1. Simplify the following.

$$(a) 1 + \sqrt[4]{3} - 4\sqrt[4]{3} = 1 - 3\sqrt[4]{3}$$

$$(b) 4\sqrt{5}(3 + \sqrt{2}) = 4\sqrt{5} \cdot 3 + 2\sqrt{5} \cdot \sqrt{2} \\ = 12\sqrt{5} + 4\sqrt{10}$$

$$(c) (1 - 3\sqrt{7})(1 + 3\sqrt{7}) = 1 - (3\sqrt{7})(3\sqrt{7}) \\ = 1 - 9 \cdot 7 \\ = 1 - 63 \\ = -62$$

2. Rationalize the denominator in each expression.

$$(a) \frac{-6}{\sqrt{18}} = \frac{-6}{\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}} = \frac{-6\sqrt{18}}{18} = \frac{-6\sqrt{9 \cdot 2}}{18} = \frac{-18\sqrt{2}}{18} = -\sqrt{2}$$

$$(b) \frac{4}{3+\sqrt{7}} = \frac{4}{3+\sqrt{7}} \cdot \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{4(3-\sqrt{7})}{9-7} = \frac{4(3-\sqrt{7})}{2} = 2(3-\sqrt{7}) = 6 - 2\sqrt{7}$$

$$(c) \frac{1}{-\sqrt{2}-\sqrt{3}}$$

$$(d) \frac{2}{\sqrt[3]{5}} = \frac{2}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{25}}{5}$$

3. Solve each equation.

$$(a) \sqrt{4t-4} - 5 = 1$$

$$\text{Isolating the radical: } \sqrt{4t-4} = 6$$

$$\text{Squaring both sides: } 4t - 4 = 36$$

$$4t = 40$$

$$t = 10$$

$$\text{Checking } t = 10: \sqrt{4 \cdot 10 - 4} = \sqrt{36} = 6 \quad \checkmark$$

$$(b) \sqrt[3]{2x+1} = 3$$

$$\text{Cubing both sides: } 2x + 1 = 27$$

$$2x = 26 \quad \Rightarrow \quad x = 13$$

$$\text{Checking } x = 13: \sqrt[3]{2(13) + 1} = \sqrt[3]{27} = 3 \quad \checkmark$$

$$(c) \ 2\sqrt{m} = \sqrt{m^2 - 3m - 8}$$

$$\begin{aligned}\text{Squaring both sides: } 4m &= m^2 - 3m - 8 \\ 0 &= m^2 - 7m - 8 \\ 0 &= (m - 8)(m + 1)\end{aligned}$$

The possible solutions are  $m = -1$  and  $m = 8$ .

Checking  $m = -1$ :  $2\sqrt{-1} = \sqrt{(-1)^2 - 3(-1) - 8}$  The left side of this equation is not real.  $\Rightarrow m = -1$  is not a solution.

Checking  $m = 8$ :  $2\sqrt{8} = \sqrt{8^2 - 3(8) - 8} = \sqrt{32}$  ✓ (The left and right side are equal.)

$$(d) \ r = \sqrt{5r + 24}$$

$$\begin{aligned}\text{Squaring both sides: } r^2 &= 5r + 24 \\ r^2 - 5r - 24 &= 0 \\ (r - 8)(r + 3) &= 0\end{aligned}$$

The possible solutions are  $r = 8$  and  $r = -3$ .

Checking  $r = 8$ :  $8 = \sqrt{5 \cdot 8 + 24} = \sqrt{64}$  ✓

Checking  $r = -3$ :  $-3 = \sqrt{5 \cdot (-3) + 24}$  The left side of this equation is not real.  $\Rightarrow r = -3$  is not a solution.