Math 98 Final Exam Practice Problem Solutions

2. Bert kicks a soccer ball up into the air with an initial velocity of 64 feet/second from the edge of a 48 foot cliff. The height of the ball above the ground below is given by $h = -16t^2 + 64t + 48$ in feet at t seconds.

   (a) When does the ball land?

   When the ball lands, the height of the ball will be 0 feet ($h = 0$). So, to find the times at which the height will be 0 feet, plug $h = 0$ into the equation and solve for t.

   $$0 = -16t^2 + 64t + 48$$
   $$0 = t^2 - 4t - 3 \quad \text{(Dividing both sides by } -16\text{)}$$

   Solving for t using the quadratic formula:

   $$t = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)} = \frac{4 \pm \sqrt{28}}{2} = \frac{4 \pm \sqrt{4 \cdot 7}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$$

   Note that $t = 2 - \sqrt{7}$ is negative and $t = 2 + \sqrt{7} \approx 4.64565$. So the ball lands at $t = 2 + \sqrt{7}$ seconds.

   (b) When does the ball reach it's maximum height? What is the maximum height?

   Since the height of the ball is described by a downward-facing parabola, the maximum height occurs at the vertex.

   The vertex formula will give us the time at which the ball is at maximum height.

   $$t = -\frac{b}{2a} = -\frac{64}{2(-16)} = -\frac{64}{-32} = 2 \text{ seconds}$$

   So, the ball is at the maximum height 2 seconds after it is kicked.

   The maximum height attained is given by

   $$h = -16(2)^2 + 64(2) + 48 = -64 + 128 + 48 = 112 \text{ feet.}$$

3. $f(x) = 2x^2 + 8x + 5$

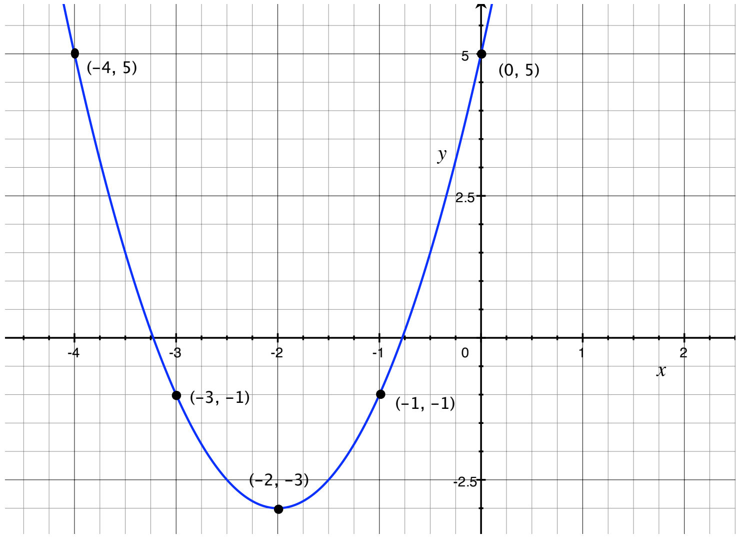
   (a) The graph of f is a parabola. What is the vertex of the parabola?

   Vertex Formula:
   x-coordinate of the vertex: $x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$

   y-coordinate of the vertex: $y = f(-2) = 2(-2)^2 + 8(-2) + 5 = -3$

   So the vertex is $(-2, -3)$.

(b) Draw the graph of f on the axis below. Include 2 points other than the vertex.



4. Evaluate the following.

   (a) $\log_5 125 =?$

   Since $5^3 = 125$, $\log_5 125 = 3$.

   (b) $\log_{11} 11 =?$

   Since $11^1 = 11$, $\log_{11} 11 = 1$. (In fact, we have that $\log_a a = 1$ for all $a > 0,\ a \neq 1$.)

   (c) $\log_4 1 =?$

   Since $4^0 = 1$, $\log_4 1 = 0$. (In fact, we have that $\log_a 1 = 0$ for all $a > 0,\ a \neq 1$.)

   (d) $\log_8 4 =?$
   Since $8^{2/3} = 4$, $\log_8 4 = \frac{2}{3}$. (Solve the equation $8^x = 4$.)

5. Fill in the following table by writing the exponential or logarithmic form for the corresponding equation.

Exponential Form	Logarithmic Form
$2^4 = 16$	$\underline{\log_2 16 = 4}$
$3^{-5} = \frac{1}{243}$	$\underline{\log_3 \frac{1}{243} = -5}$
$\underline{7^3 = 343}$	$\log_7 343 = 3$
$\underline{10^{-2} = \frac{1}{100}}$	$\log \frac{1}{100} = -2$

6. Find the inverses for the following functions.

(a) $f(x) = \sqrt{x+1}$ (Be sure to consider the domain and range of the function and its inverse.)

$$y = \sqrt{x+1}$$

Switching x and y: $x = \sqrt{y+1}$

Solving for y: $x^2 = y + 1$
$\Rightarrow \quad y = x^2 - 1$

Note: The domain of $f(x)$ is $x \geq -1$ and the range is $y \geq 0$. So, the inverse function is $f^{-1}(x) = x^2 - 1$ with the domain $x \geq 0$ and range $y \geq -1$. (It is the right half of the parabola $x^2 - 1$.)

(b) $g(x) = 2x^3 - 4$

$$y = 2x^3 - 4$$

Switching x and y: $x = 2y^3 - 4$

Solving for y: $x + 4 = 2y^3$
$\frac{1}{2}x + 2 = y^3$ Dividing both sides by 2.
$\Rightarrow \quad y = \sqrt[3]{\frac{1}{2}x + 2}$ Taking the cube root of each side.

So, $g^{-1}(x) = \sqrt[3]{\frac{1}{2}x + 2}$ and the domain and range for the function g and its inverse are $(-\infty, \infty)$.

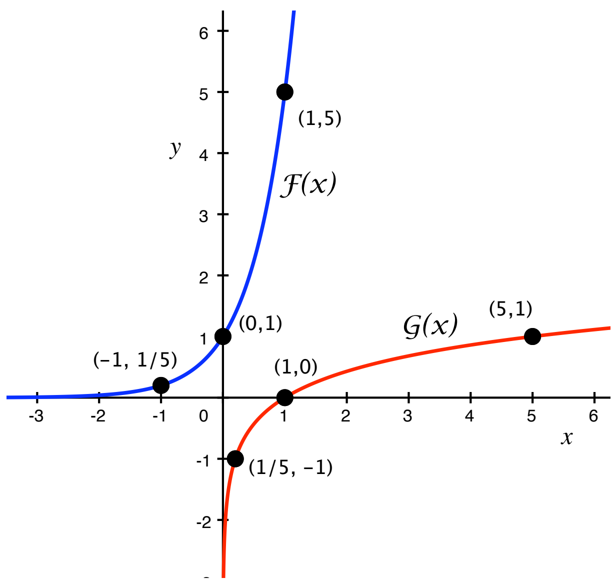
(c) $h(x) = \log_5 x$

$$y = \log_5 x \quad \Rightarrow \quad 5^y = x$$

Switching x and y: $5^x = y$

So, $h^{-1}(x) = 5^x$. The domain of h is $(-\infty, \infty)$ and the range is $(0, \infty)$. The domain of h^{-1} is $(0, \infty)$ and the range is $(-\infty, \infty)$.

7. Graph the functions $F(x) = 5^x$ and $G(x) = \log_5 x$.



8. Solve the following equations for the variable x.

(a) $4^x = \frac{1}{16}$

Since $\frac{1}{16} = 4^{-2}$, we can modify the right side of the original equation to get:

$$4^x = 4^{-2}$$

Both expressions have the same base. $\Rightarrow$ The exponents are equal. $\Rightarrow$ $x = -2$

(b) $9^{6x} = 27^{x+1}$

Both 9 and 27 are powers of 3 $(9 = 3^2,\ 27 = 3^3)$, so $9^{6x} = 27^{x+1}$

$$\Rightarrow \quad (3^2)^{6x} = (3^3)^{x+1}$$
$$\Rightarrow \quad 3^{12x} = 3^{3x+3}$$
$$\Rightarrow \quad 12x = 3x + 3$$
$$\Rightarrow \quad x = \frac{1}{3}$$

(c) $\log_8 x = 2$

Rewriting this equation in exponential form, we have $8^2 = x$ or $x = 64$.

(d) $\log_x 216 = 3$

Rewriting this equation in exponential form, we have $x^3 = 216$

$$\Rightarrow x = 216^{1/3} = 6.$$

9. You fire a cannonball upward so that its distance (in feet) above the ground t seconds after firing is given by $h(t) = -16t^2 + 144t$. Find the maximum height it reaches and the number of seconds it takes to reach that height.

Note that the graph of the function h that describes the height of the cannonball is a downward-facing parabola. Thus, the maximum height of the cannonball occurs at the vertex of the parabola.

Using the vertex formula, we have that $t = \frac{-b}{2a} = \frac{-144}{2(-16)} = 4.5$ seconds.
So, the cannonball reaches it maximum height 4.5 seconds after it is fired and the maximum height is $h(4.5) = -16(4.5)^2 + 144(4.5) = 324$ feet.

10. A (fictitious) survey of the squirrel population on the Shoreline campus is conducted and it is found that 1200 squirrels are present on campus this quarter. It is projected that the population of squirrels on campus can be described by the following exponential function: $s(t) = 1200(3)^t$ where t is the number of years after the survey is conducted.

(a) How many squirrels can we expect to have on campus 8 years after the survey?

$$\begin{aligned}\text{Predicted number of squirrels after 8 years} = s(8) &= 1200(3)^8 \\ &= 1200(6561) \\ &= 7,873,200 \text{ squirrels}\end{aligned}$$

(b) When can we expect to have 97200 squirrels on campus?

We are trying to find a time t for which $s(t) = 97200$.

$$\begin{aligned}\text{Setting } s(t) \text{ equal to 97200} \Rightarrow \quad 97200 &= 1200(3)^t \\ 81 &= 3^t \quad \text{(Dividing both sides by 1200)} \\ 3^4 &= 3^t \quad \text{(Since } 3^4 = 81) \\ \Rightarrow \quad t &= 4\end{aligned}$$

The squirrel population is expected to be 97200 after 4 years.