Math 98
Final Exam Solutions

1. (8 pts.) Simplify the following.

   (a) (4 pts.) $27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2 = 3^2 = 9$

   (b) (4 pts.) $\sqrt{18x^7} = \sqrt{9 \cdot 2 \cdot x^6 \cdot x} = 3x^3\sqrt{2x}$

2. (6 pts.) Find the inverse function of $F(x) = \frac{1}{3}x - 2$.

   We have $y = \frac{1}{3}x - 2$.

   Switching x and y: $x = \frac{1}{3}y - 2$

   Solving for y:
   $$x + 2 = \frac{1}{3}y$$
   $$3(x+2) = y$$
   $$\Rightarrow \quad y = 3x + 6$$

   So, $F^{-1}(x) = 3x + 6$.

3. (11 pts.) $f(x) = -2x^2 + 8x - 5$

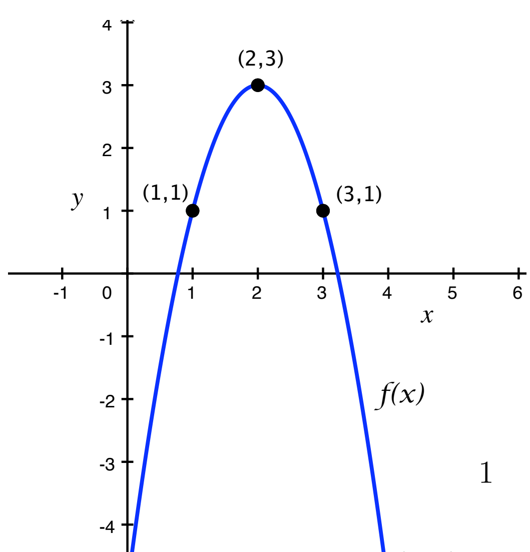
   (a) (6 pts.) Find the vertex of the graph of f.

   x-coordinate of vertex: $x = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2$

   y-coordinate of vertex: $y = f(2) = -2(2)^2 + 8(2) - 5 = -8 + 16 - 5 = 3$

   So, the vertex is $(2, 3)$.

   (b) (5 pts.) Sketch the graph of f below. Include 2 points other than the vertex and the domain and range.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

4. (9 pts.) Find the distance between the points $(2, \sqrt{3})$ and $(-2, 3\sqrt{3})$. Simplify your answer as much as possible.

$$\begin{aligned} \text{Distance} = \sqrt{(-2-2)^2 + (3\sqrt{3} - \sqrt{3})^2} &= \sqrt{4^2 + (2\sqrt{3})^2} \\ &= \sqrt{16 + 4^2(\sqrt{3})^2} \\ &= \sqrt{16+12} \\ &= \sqrt{28} \\ &= \sqrt{4 \cdot 7} \\ &= 2\sqrt{7} \end{aligned}$$

5. (18 pts.) Solve the following equations.

   (a) (4 pts.) $9^t = \frac{1}{3}$

   Here are two ways to do this problem.

   - Note that $9^{1/2} = 3$. Then $9^{-1/2} = \frac{1}{3}$. So, $t = -\frac{1}{2}$.

   - Rewriting with the same base on both sides:

   $$\begin{aligned} 9^t = \tfrac{1}{3} \quad \Rightarrow \quad (3^2)^t = 3^{-1} \quad &\Rightarrow \quad 3^{2t} = 3^{-1} \\ &\Rightarrow \quad 2t = -1 \\ &\Rightarrow \quad t = -\tfrac{1}{2} \end{aligned}$$

   (b) (5 pts.) $49^{x-\frac{1}{2}} = 7^{x+1}$

   Rewriting with the same base on both sides:

   $$\begin{aligned} 49^{x-\frac{1}{2}} = 7^{x+1} \quad \Rightarrow \quad (7^2)^{x-\frac{1}{2}} = 7^{x+1} \quad &\Rightarrow \quad 7^{2x-1} = 7^{x+1} \\ &\Rightarrow \quad 2x - 1 = x + 1 \\ &\Rightarrow \quad x = 2 \end{aligned}$$

   (c) (5 pts.) $\log_a 64 = 3$

   In exponential form: $a^3 = 64 \quad \Rightarrow \quad a = 4$

   (d) (4 pts.) $\log_5 m = 3$

   In exponential form: $5^3 = m \quad \Rightarrow \quad m = 125$

6. (15 pts.) You have a lovely garden full of eggplants so you decide to build a eggplant catapult. On your test run, you find that the height of the launched eggplant in feet is given by $h(t) = -16t^2 + 112t$ at t seconds.

(a) (3 pts.) How high is the eggplant after 2 seconds?

The height of the eggplant at $t = 2$ seconds is given by
$$h(2) = -16(2)^2 + 112(2) = -64 + 224 = 160 \text{ feet.}$$

(b) (6 pts.) What is the maximum height that the eggplant attains? When does the eggplant reach this maximum height?

The eggplant attains its maximum height when $t = \frac{-b}{2a} = \frac{-112}{2(-16)} = 3.5$ seconds.

The maximum height attained is given by
$$h(3.5) = -16(3.5)^2 + 112(3.5) = -196 + 392 = 196 \text{ feet.}$$

(c) (6 pts.) At what time(s) will the eggplant reach a height of 52 feet?

We are trying to find t such that $h(t) = -16t^2 + 112t = 52$.

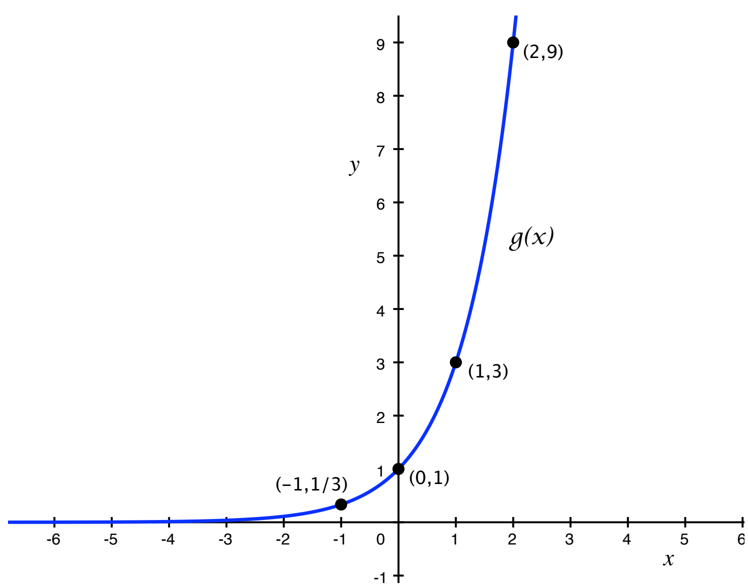
$$\Rightarrow \quad -16t^2 + 112t - 52 = 0$$
$$4t^2 - 28t + 13 = 0 \quad \text{(Dividing both sides by } -4.)$$

This can be solved by factoring (or by using the quadratic formula). When factored, we have $(2t - 13)(2t - 1) = 0 \quad \Rightarrow \quad t = \frac{13}{2} = 6.5$ seconds or $t = \frac{1}{2} = 0.5$ seconds.

So, the eggplant is 52 feet high at 0.5 seconds and at 6.5 seconds.

7. (10 pts.) $g(x) = 3^x$

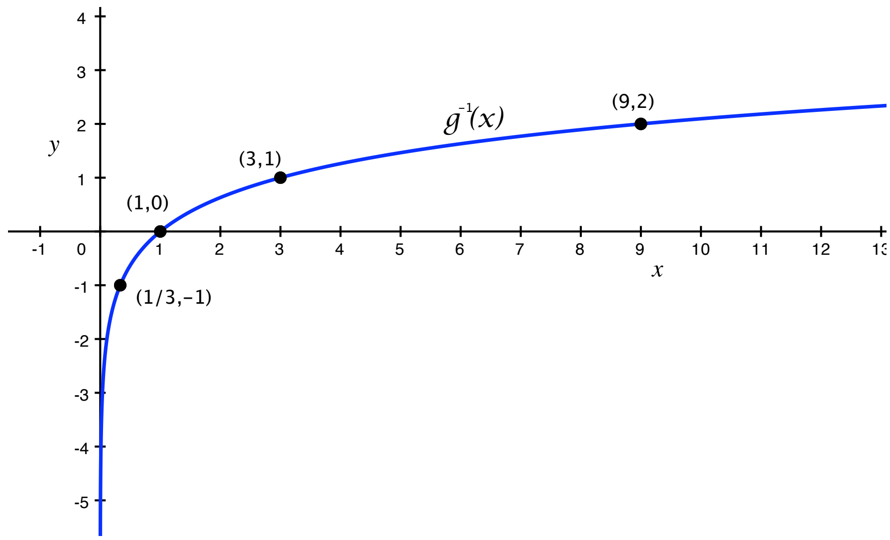
(a) (5 pts.) Sketch the graph of g below. Include at least 3 points and the domain and range.



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

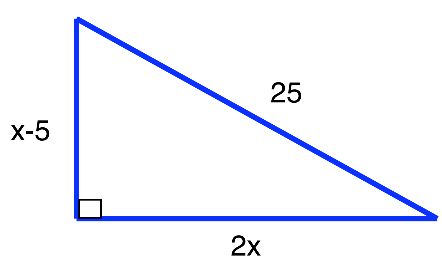
(b) (5 pts.) Sketch the graph of g^{-1} below. Include at least 3 points and the domain and range.



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

8. (11 pts.) Find the lengths of the sides of the following right triangle.



By Pythagorean Theorem:

$$(2x)^2 + (x-5)^2 = 25^2$$

Simplifying:

$$4x^2 + x^2 - 10x + 25 = 625$$
$$5x^2 - 10x - 600 = 0$$
$$x^2 - 2x - 120 = 0 \quad \text{(Dividing both sides by 5.)}$$

This can be solved by factoring (or using the quadratic formula). When factored, we have $(x-12)(x+10) = 0 \quad \Rightarrow \quad x = 12$ or $x = -10$.

If $x = -10$, then we would have negative lengths for the sides of the triangle, so we cannot have $x = -10$ be a solution.

If $x = 12$, then the lengths of the sides of the triangle are 24, 7, and 25.