# Math 98 Practice Problems Solutions for Exam #2

2. Solve the following equations.

   (a) $2\sqrt[3]{3x+1}+4=0$

   Isolating the radical:
   $$2\sqrt[3]{3x+1} = -4$$
   $$\sqrt[3]{3x+1} = -2$$

   Cubing both sides:
   $$3x+1=-8$$
   $$\Rightarrow \quad 3x=-9 \quad \Rightarrow \quad x=-3$$

   $\Rightarrow \quad x=-3$ is a possible solution to the original equation.

   **Check:** $2\sqrt[3]{3(-3)+1}+4 = 2\sqrt[3]{-8}+4 = -4+4=0$ ✓

   So, $x=-3$ is a solution.

   (b) $\sqrt{5+t}=t+3$

   Squaring both sides:
   $$5+t=(t+3)^2=(t+3)(t+3)$$
   $$\Rightarrow \quad 5+t=t^2+6t+9$$
   $$0=t^2+5t+4$$
   $$0=(t+4)(t+1)$$

   $\Rightarrow \quad t=-4$ or $t=-1$ are possible solutions.

   **Check:** $\underline{t=-4}$ $\quad \sqrt{5-4} \neq -4+3$

   $\underline{t=-1}$ $\quad \sqrt{5-1}=-1+3$ ✓

   So, $t=-1$ is the solution of the original equation.

   (c) $\dfrac{3}{x}+4=\dfrac{1}{x+2}$

   Multiplying both sides by $x(x+2)$:
   $$x(x+2)(\tfrac{3}{x}+4)=x(x+2)(\tfrac{1}{x+2})$$
   $$x(x+2)\tfrac{3}{x}+4x(x+2)=x$$
   $$3(x+2)+4x(x+2)=x$$
   $$3x+6+4x^2+8x=x$$
   $$4x^2+10x+6=0$$
   $$2x^2+5x+3=0 \quad \text{(Dividing by 2)}$$
   $$(2x+3)(x+1)=0$$
   $$\Rightarrow \quad x=-\tfrac{2}{3} \quad \text{or} \quad x=-1$$

(d) $z^4 - 32 = 14z^2$

Putting all terms on one side: $z^4 - 14z^2 - 32 = 0$

We can factor this into the following: $(z^2 - 16)(z^2 + 2) = 0$

$$\begin{aligned} &\Rightarrow \quad z^2 = 16 \quad \text{or} \quad z^2 = -2 \\ &\Rightarrow \quad z = \pm\sqrt{16} \quad \text{or} \quad z = \pm\sqrt{-2} \\ &\Rightarrow \quad z = \pm 4 \quad \text{or} \quad z = \pm i\sqrt{2} \end{aligned}$$

So, we have four solutions: $z = 4,\ -4,\ i\sqrt{2},\ -i\sqrt{2}$

3. Carry out the following operations and simplify as much as possible.

(a) $(5 - 3i) - (2 + i)$

$$(5 - 3i) - (2 + i) \ = \ 5 - 3i - 2 - i \ = \ 3 - 4i$$

(b) $(5 - 3i) \cdot (2 + i)$

$$\begin{aligned} (5 - 3i) \cdot (2 + i) &= 10 + 5i - 6i - 3i^2 \qquad \text{(FOIL)} \\ &= 10 - i - 3(-1) \qquad (i^2 = -1) \\ &= 13 - i \end{aligned}$$

4. You launch a toy rocket with an initial velocity of 120 feet/second. The height of the rocket is given by $h = -16t^2 + 120t$ in feet at $t$ seconds.

(a) When does the rocket land?

When the rocket lands, the height of the rocket will be 0 feet, so we are trying to find a value of $t$ so that $h = 0$.

Setting $h = 0$ and solving for $t$:

$$\begin{aligned} 0 &= -16t^2 + 120t \\ 0 &= -8t(2t - 15) \qquad \text{(Factoring out } -8t) \end{aligned}$$

By the zero-factor property, we have that either $-8t = 0$ or $2t - 15 = 0$.
So, either $t = 0$ seconds or $t = 7.5$ seconds.

Since the rocket launches at 0 seconds, it must land at 7.5 seconds.

(b) How high is the rocket after 3 seconds?

We are looking for the height ($h =$?) when $t = 3$.

Setting $t = 3$ and solving for $h$: $$h = -16(3)^2 + 120(3)$$ $$= -16(9) + 120(3) = 216 \text{ feet}$$

So, the height is 216 feet at 3 seconds.

(c) Find all the times at which the rocket is 144 feet high.

We are looking for the time ($t =$?) at which $h = 144$.

Setting $h = 144$ and solving for $t$: $$144 = -16t^2 + 120t$$ $$0 = -16t^2 + 120t - 144$$ $$0 = 2t^2 - 15t + 18$$ (Dividing both sides by $-8$)

Solving the equation (Here are two ways):

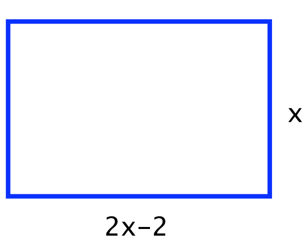
- Factoring: $0 = (2t - 3)(t - 6)$
  $\Rightarrow \quad 2t - 3 = 0 \quad \text{or} \quad t - 6 = 0$
  $\Rightarrow \quad t = 1.5 \quad \text{or} \quad t = 6$

  So, the rocket is 144 feet high at 1.5 seconds and 6 seconds.

- Quadratic Formula: $a = 2,\ b = -15,\ c = 18$
  $$t = \frac{15 \pm \sqrt{(-15)^2 - 4(2)(18)}}{2(2)} = \frac{15 \pm \sqrt{225 - 144}}{4} = \frac{15 \pm \sqrt{81}}{4}$$ $$= \frac{15 \pm 9}{4}$$ $$= \frac{24}{4} \text{ or } \frac{6}{4}$$ $$= 6 \text{ or } 1.5$$

  So, the rocket is 144 feet hight at 1.5 seconds and 6 seconds.

5. The following rectangle has an area of 84 square units. What are the dimensions of the rectangle given the lengths in the diagram below?




Width of rectangle $= x$
Length of rectangle $= 2x - 2$
$\Rightarrow$ Area of rectangle $= x(2x - 2)$

$$\Rightarrow \quad 84 = x(2x - 2)$$ $$84 = 2x^2 - 2x$$ $$0 = 2x^2 - 2x - 84$$ $$0 = x^2 - x - 42$$

Factoring this equation: $0 = (x-7)(x+6)$

$$\Rightarrow \quad x-7=0 \quad \text{or} \quad x+6=0$$
$$\Rightarrow \quad x=7 \quad \text{or} \quad x=-6$$

Since lengths are positive, the only solution that makes sense in this application is $x = 7$. If $x = 7$, then the dimensions of the rectangle are 7 by 12 units.

6.

(a) Use the discriminant to determine the number and type (rational, irrational, or imaginary) of solutions of the equation $x^2 - 6x + 13 = 0$.

Discriminant $= b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16 < 0$

Since the discriminant is negative, we have 2 imaginary solutions to the equation.

(b) Use the quadratic formula to solve the equation $x^2 - 6x + 13 = 0$.

Solutions: $$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

7. Solve the following equations. Simplify your answer as much as possible.

(a) $(3m+6)^2 + 4 = 1$

$(3m+6)^2 + 4 = 1 \quad \Rightarrow \quad (3m+6)^2 = -3$

Using the square root property, we have that $3m + 6 = \pm\sqrt{-3} = \pm i\sqrt{3}$

$$\Rightarrow \quad 3m = -6 \pm i\sqrt{3}$$
$$\Rightarrow \quad m = \frac{-6 \pm i\sqrt{3}}{3}$$

(b) $4x^2 + 22x = 12$ (Solve by completing the square.)

$x^2 + \frac{11}{2}x = 3$ (Dividing both sides by 4)

Note that $(x + \frac{11}{4})^2 = x^2 + \frac{11}{2}x + (\frac{11}{4})^2 = x^2 + \frac{11}{2}x + \frac{121}{16}$.

Adding $\frac{121}{16}$ to both sides of the equation:

$$x^2 + \frac{11}{2}x + \frac{121}{16} = 3 + \frac{121}{16} = \frac{48}{16} + \frac{121}{16} = \frac{169}{16}$$

$$(x + \tfrac{11}{4})^2 = \frac{169}{16}$$
$$x + \frac{11}{4} = \pm\sqrt{\frac{169}{16}} = \pm\frac{13}{4}$$

$$x = -\tfrac{11}{4} \pm \tfrac{13}{4} = \tfrac{1}{2} \quad \text{or} \quad -6$$