Math 98 Practice Problems Solutions for Exam #1

1. Simplify the following as much as possible.

   (a) $\sqrt{\frac{m^6}{81}} = \frac{m^3}{9}$

   (b) $2a^{2/3}(a^{4/3} - 3a^{-5/3})$

   $$\begin{aligned} 2a^{2/3}(a^{4/3} - 3a^{-5/3}) &= 2a^{2/3} \cdot a^{4/3} - 6a^{2/3} \cdot a^{-5/3} \\ &= 2a^{6/3} - 6a^{-3/3} \\ &= 2a^2 - 6a^{-1} \\ &= 2a^2 - \frac{6}{a} \end{aligned}$$

   (c) $\sqrt[4]{r^3} \cdot \sqrt[8]{r^3}$ (Leave your answer in exponential form.)

   $$\sqrt[4]{r^3} \cdot \sqrt[8]{r^3} = r^{3/4} \cdot r^{3/8} = r^{6/8} \cdot r^{3/8} = r^{9/8}$$

   (d) $\sqrt[3]{16x^6y^{11}} = 2x^2y^3\sqrt[3]{2y^2}$

   (e) $$\begin{aligned} 4 - \sqrt{6} + 3\sqrt{54} &= 4 - \sqrt{6} + 3\sqrt{9 \cdot 6} \\ &= 4 - \sqrt{6} + 9\sqrt{6} \\ &= 4 + 8\sqrt{6} \end{aligned}$$

   (f) $\frac{\sqrt{27}}{2+\sqrt{3}}$ (Rationalize the denominator and simplify.)

   $$\frac{\sqrt{27}}{2+\sqrt{3}} = \frac{3\sqrt{3}}{2+\sqrt{3}} = \frac{3\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3\sqrt{3}(2-\sqrt{3})}{4-3} = 3\sqrt{3}(2-\sqrt{3}) = 6\sqrt{3} - 3 \cdot 3 = 6\sqrt{3} - 9$$

   (g) $\sqrt[6]{y^5} \cdot \sqrt[3]{y^2}$ (Give answer in exponential form.)

   $$\sqrt[6]{y^5} \cdot \sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{5/6+4/6} = y^{9/6} = y^{3/2}$$

   (h) $\left(\frac{a^{-2}b^3}{a^4b^{-3}}\right)^{-\frac{2}{3}}$ (Give answer with positive exponents only.)

   $$\left(\frac{a^{-2}b^3}{a^4b^{-3}}\right)^{-\frac{2}{3}} = \left(\frac{b^3b^3}{a^4a^2}\right)^{-\frac{2}{3}} = \left(\frac{b^{3+3}}{a^{4+2}}\right)^{-\frac{2}{3}} = \left(\frac{b^6}{a^6}\right)^{-\frac{2}{3}} = \left(\frac{a^6}{b^6}\right)^{\frac{2}{3}} = \frac{a^{6(\frac{2}{3})}}{b^{6(\frac{2}{3})}} = \frac{a^4}{b^4}$$

   (i) $-\sqrt{100x^6y^4}$

   $$-\sqrt{100x^6y^4} = -10x^3y^2 \text{ since } \sqrt{100} = 10,\ \sqrt{x^6} = x^3,\ \sqrt{y^4} = y^2$$

   (j) $2\sqrt{18a^3} + 5a\sqrt{72a}$

$$2\sqrt{18a^3} + 5a\sqrt{72a} = 2\sqrt{9 \cdot 2a^2 \cdot a} + 5a\sqrt{36 \cdot 2a} = 2 \cdot 3a\sqrt{2a} + 5a \cdot 6\sqrt{2a}$$
$$= 6a\sqrt{2a} + 30a\sqrt{2a}$$
$$= 36a\sqrt{2a}$$

(k) $\frac{\sqrt{8}}{2-\sqrt{2}}$ (Rationalize the denominator and simplify.)

$$\frac{\sqrt{8}}{2-\sqrt{2}} = \frac{\sqrt{8}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{8}(2+\sqrt{2})}{4-2} \text{ since } (2+\sqrt{2})(2-\sqrt{2}) = 2^2-(\sqrt{2})^2 = 4-2$$
$$= \frac{2\sqrt{8}+\sqrt{16}}{2}$$
$$= \frac{4\sqrt{8}+4}{2} = 2\sqrt{2}+2$$

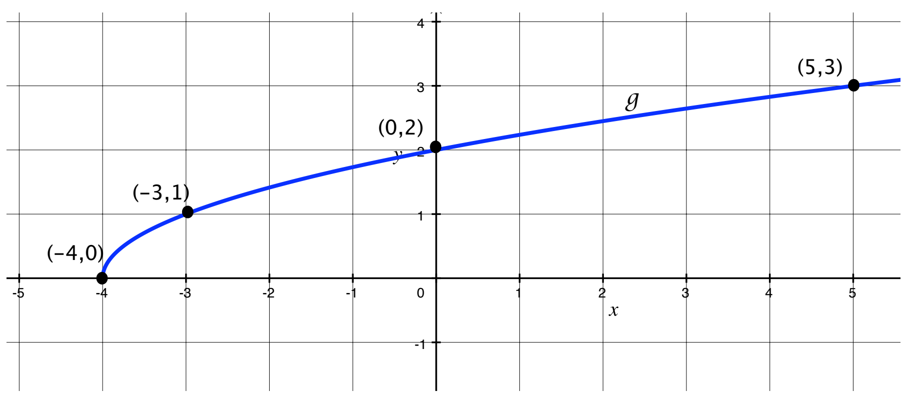
(l) $\sqrt{\frac{9}{x^6}}+\sqrt{\frac{1}{25}}$ (Write the answer as one fraction.)

$$\sqrt{\frac{9}{x^6}}+\sqrt{\frac{1}{25}} = \frac{\sqrt{9}}{\sqrt{x^6}}+\frac{\sqrt{1}}{\sqrt{25}} = \frac{3}{x^3}+\frac{1}{5} = \frac{15}{5x^3}+\frac{x^3}{5x^3} = \frac{15+x^3}{5x^3}$$

2. Graph $g(x) = \sqrt{x+4}$ and state the domain and range of g.

Finding points on the graph:

x	y
-4	$\sqrt{-4+4} = 0$
-3	$\sqrt{-3+4} = 1$
0	$\sqrt{0+4} = 2$
5	$\sqrt{5+4} = 3$



Domain: $[-4, \infty)$ or $x \geq -4$ **Range:** $[0, \infty)$ or $y \geq 0$

3. (10 pts.) Find the distance between the points $(1, -3)$ and $(-4, 2)$. Simplify your answer.

$$d = \sqrt{(1-(-4))^2 + (-3-2)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

4. Find the distance between the points $(\sqrt{6}, 2)$ and $(-\sqrt{6}, -1)$.

Using the distance formula, we have that the distance is

$$\begin{aligned}\text{Distance} = \sqrt{(-\sqrt{6} - \sqrt{6})^2 + (-1 - 2)^2} &= \sqrt{(-2\sqrt{6})^2 + (-3)^2} \\ &= \sqrt{4 \cdot 6 + 9} \\ &= \sqrt{33}\end{aligned}$$