Math 80
Exam 2 Solutions

1. (15 pts.) Find an equation for a line through $(-3, 1)$ that is perpendicular to the line $2y = -6x + 8$. Write the equation in slope-intercept form.

   Note that $2y = -6x + 8 \quad \Rightarrow \quad y = -3x + 4$ (Dividing both sides by 2).
   So, the line $2y = -6x + 8$ has slope -3 (coefficient of x).

   We want a line that is perpendicular to $2y = -6x + 8$, so the slope of our line is $\frac{1}{3}$ (negative reciprocal).

   Using the point-slope form of a linear equation, we have $y - 1 = \frac{1}{3}(x - (-3))$
   $\Rightarrow \quad y - 1 = \frac{1}{3}(x + 3)$

   Solving for y and simplifying will put this equation into slope-intercept form.
   $\Rightarrow \quad y = \frac{1}{3}x + 2$

2. (10 pts.) The mass of one water molecule is approximately 3×10^{-23} grams. What is the mass of 2×10^{25} water molecules? (Use scientific notation in your calculations.)

   Since one water molecule is 3×10^{-23} grams, the mass of 2×10^{25} molecules is
   $$(3 \times 10^{-23}) \cdot (2 \times 10^{25}) = (3 \cdot 2) \times (10^{-23} \cdot 10^{25}) = 6 \times 10^{2} = 600 \text{ grams.}$$

3. (25 pts.) Simplify the following exponential expressions as much as possible. Write answers with positive exponents only.

   (a) (8 pts.) $\frac{4^{-2}}{2^{-3}} \cdot (-3^0)$

   $$\frac{4^{-2}}{2^{-3}} \cdot (-3^0) = \frac{2^3}{4^2} \cdot (-1) = -\frac{8}{16} = -\frac{1}{2}$$

   (b) (8 pts.) $(ab^2)^4 \cdot (2a^6bc)$

   $$(ab^2)^4 \cdot (2a^6bc) = (a^4b^8) \cdot (2a^6bc) = 2a^4a^6b^8bc = 2a^{10}b^9c$$

   (c) (9 pts.) $\frac{(3x^{-3}y^2)^{-2}}{x^5y^{-5}}$

   $$\frac{(3x^{-3}y^2)^{-2}}{x^5y^{-5}} = \frac{3^{-2}x^6y^{-4}}{x^5y^{-5}} = 3^{-2}xy^{-9} \text{ (Quotient Rule)}$$
   $$= \frac{x}{3^2y^9} = \frac{x}{9y^9}$$

Note: There are different ways to use the exponential rules to simplify the above expressions. However, the final answers will be the same.

4. (25 pts.) Carry out the following operations and simplify as much as possible.

   (a) (8 pts.) Subtract $-2a^7 + a^4 - 3$ from the polynomial $5a^7 - 3a^4 + a$.

   $$\begin{aligned}(5a^7 - 3a^4 + a) - (-2a^7 + a^4 - 3) &= 5a^7 - 3a^4 + a + 2a^7 - a^4 + 3 \\ &= 7a^7 - 4a^4 + a + 3 \text{ (Combining like terms)}\end{aligned}$$

   (b) (9 pts.) $3(m+1)(n^2m - n^2 + 2)$

   $$\begin{aligned}3(m+1)(n^2m - n^2 + 2) &= 3[m(n^2m - n^2 + 2) + 1(n^2m - n^2 + 2)] \\ &= 3[n^2m^2 - n^2m + 2m + n^2m - n^2 + 2] \\ &= 3[n^2m^2 + 2m - n^2 + 2] \\ &= 3n^2m^2 + 6m - 3n^2 + 6\end{aligned}$$

   (c) (8 pts.) Divide $3y^5z^2 - 6y^3z + 3y^2z^2$ by the monomial $-3y^2z^2$.

   $$\begin{aligned}\frac{3y^5z^2 - 6y^3z + 3y^2z^2}{-3y^2z^2} &= \frac{3y^5z^2}{-3y^2z^2} + \frac{-6y^3z}{-3y^2z^2} + \frac{3y^2z^2}{-3y^2z^2} \\ &= -y^3 + \frac{2y}{z} - 1 \text{ (Quotient Rule)}\end{aligned}$$

5. (25 pts.) Factor the following polynomials **as much as possible**.

   (a) (8 pts.) $st - 2s + 5t^2 - 10t$ (Factor by grouping)

   Note: The GCF of st and $-2s$ is s and the GCF of $5t^2$ and $-10t$ is $5t$.

   $$\begin{aligned}st - 2s + 5t^2 - 10t &= s(t-2) + 5t(t-2) \\ &= (s + 5t)(t - 2)\end{aligned}$$

   (b) (8 pts.) $a^2 - 3a - 40$

   Since the constant term -40 is negative, we need to consider the following pairs of factors:

Pairs of Factors	Product	Sum
-1 & 40	-40	11
1 & -40	-40	-11
-2 & 20	-40	4
2 & -20	-40	-4
-4 & 10	-40	1
4 & -10	-40	-1
-5 & 8	-40	3
5 & -8	-40	-3

Since the sum of 5 and -8 is -3, we know that $a^2 - 3a - 40 = (a + 5)(a - 8)$.

(c) (9 pts.) $2x^4y + 18x^3y - 20x^2y$

Note that the GCF of all the terms is $2x^2y$.
Factoring out the GCF: $2x^4y + 18x^3y - 20x^2y = 2x^2y(x^2 + 9x - 10)$

The polynomial $x^2 + 9x - 10$ can be factored further into $(x + 10)(x - 1)$.
(Factors of -10: -1 & 10, 1 & -10, -2 & 5, 2 & -5)

So, $2x^4y + 18x^3y - 20x^2y = 2x^2y(x + 10)(x - 1)$.