Math 80 Quiz 6 Solutions

1. Factor the following.

   (a) $2t^2 - 13t + 18 = (2t-9)(t-2)$

   To factor this using **Method 1** as described in Section 5.3, we first look at the product of 2 (the coefficient of t^2) and 18 (the constant term), which is $2 \cdot 18 = 36$. Then we look at factors of 36 to get two integers whose product is 36 and whose sum is -13. We have the following pairs of factors:
   $(-1,-36),\ (-2,-18),\ (-3,-12),\ (-4,-9),\ (-6,-6)$.
   Of these pairs, only -4 and -9 add up to -13.

   So, we use these to split up the original trinomial into a polynomial that we can factor by grouping.

   $$\begin{aligned} 2t^2 - 13t + 18 = 2t^2 - 4t - 9t + 18 &= (2t^2 - 4t) + (-9t + 18) \\ &= 2t(t-2) - 9(t-2) \text{ (Factoring the GCF from each group.)} \\ &= (2t-9)(t-2) \text{ (Factoring out (t-2) from each term.)} \end{aligned}$$

   Using **Method 2**, you would note that the factors of 2 are 1 and 2, so to get the $2t^2$ term, you would have to have $(2t \quad)(t \quad)$.
   Then looking at the factors of 18 that would give us a negative middle term $(-13t)$ we have the following pairs to consider:
   $(-1,-18),\ (-2,-9),\ (-3,-6)$

   FOILing out the following possible answers:

   $$(2t-1)(t-18),\ (2t-18)(t-1),\ (2t-2)(t-9),\ (2t-9)(t-2),$$
   $$(2t-3)(t-6),\ (2t-6)(t-3),$$

   we see that only $(2t-9)(t-2)$ gives us a middle term of $-13t$.

   (b) $-5x^2 + 2x + 16 = -1(5x+8)(x-2)$ OR $(5x+8)(2-x)$ OR $(-5x-8)(x-2)$

   To make this simpler, we will factor out a -1 from the entire polynomial:
   $-5x^2 + 2x + 16 = -1(5x^2 - 2x - 16)$. Now we just need to factor $5x^2 - 2x - 16$.

   To factor this using **Method 1**, we first look at the product of 5 (the coefficient of t^2) and -16 (the constant term), which is $5 \cdot -16 = -80$. Then we look at factors of -80 to get two integers whose product is -80 and whose sum is -2. We have the following pairs of factors:
   $(-1,80),\ (1,-80),\ (-2,40),\ (2,-40),\ (-5,16),\ (5,-16),\ (-8,10),\ (8,-10)$.
   Of these pairs, only 8 and -10 add up to -2.

   So, we use these to split up the original trinomial into a polynomial that we can factor by grouping.

$$5x^2 - 2x - 16 = 5x^2 - 10x + 8x - 16 = (5x^2 - 10x) + (8x - 16)$$
$$= 5x(x-2) + 8(x-2) \text{ (Factoring the GCF from each group.)}$$
$$= (5x+8)(x-2) \text{ (Factoring out (x-2) from each term.)}$$

So $-5x^2 + 2x + 16 = -1(5x^2 - 2x - 16) = -1(5x+8)(x-2)$

Using **Method 2**, you would note that the factors of 5 are 1 and 5, so to get the $5x^2$ term, you would have to have $(5x \quad)(x \quad)$.
Then looking at the factors of -16 we have the following pairs to consider:
$(-1, 16),\ (1, -16),\ (-2, 8),\ (2, -8),\ (-4, 4)$

FOILing out the following possible answers, we see that only $(5x+8)(x-2)$ gives us a middle term of $-2x$.

So $-5x^2 + 2x + 16 = -1(5x^2 - 2x - 16) = -1(5x+8)(x-2)$

(c) $a^2 - 81 = (a+9)(a-9)$

This is a difference of squares as featured in special factoring rules (Sect. 5.4). Note that $a^2 = (a)^2$ and $81 = 9^2$. So, $a^2 - 81 = (a+9)(a-9)$

(d) $(2x^2 + 5x)(x-1)^2 - 12(x-1)^2 = (x-1)^2(2x-3)(x+4)$

The very first step is to factor out the common factor $(x-1)^2$ from each term. This leaves us with
$(2x^2+5x)(x-1)^2 - 12(x-1)^2 = (x-1)^2((2x^2+5x) - 12) = (x-1)^2(2x^2+5x-12)$

$(x-1)^2$ is already factored, so now we have to factor $2x^2 + 5x - 12$.

To factor this using **Method 1**, we first look at the product of 2 (the coefficient of x^2) and -12 (the constant term), which is $2 \cdot -12 = -24$. Then we look at factors of -24 to get two integers whose product is -24 and whose sum is 5. We have the following pairs of factors:
$(-1, 24),\ (1, -24),\ (-2, 12),\ (2, -12),\ (-3, 8),\ (3, -8),\ (-4, 6),\ (4, -6)$.
Of these pairs, only -3 and 8 add up to 5.

So, we use these to split up the original trinomial into a polynomial that we can factor by grouping.

$$2x^2 + 5x - 12 = 2x^2 + 8x - 3x - 12 = (2x^2 + 8x) + (-3x - 12)$$
$$= 2x(x+4) - 3(x+4) \text{ (Factoring the GCF from each group.)}$$
$$= (2x-3)(x+4) \text{ (Factoring out (x+4) from each term.)}$$

So $(2x^2+5x)(x-1)^2 - 12(x-1)^2 = (x-1)^2(2x^2+5x-12) = (x-1)^2(2x-3)(x+4)$

2. Solve the following equations.

(a) $2t^2 - 13t + 18 = 0 \quad (t = \frac{9}{2} \quad OR \quad t = 2)$

Using the fact that $2t^2 - 13t + 18 = (2t-9)(t-2)$ from problem 1a, we have that

$$\begin{aligned} & 2t^2 - 13t + 18 = 0 \\ \Rightarrow\ & (2t-9)(t-2) = 0 \\ \Rightarrow\ & 2t - 9 = 0 \quad OR \quad t - 2 = 0 \\ \Rightarrow\ & t = \tfrac{9}{2} \quad OR \quad t = 2 \end{aligned}$$

(b) $-5x^2 + 2x + 16 = 0 \quad (x = \frac{-8}{5} \quad OR \quad x = 2)$

Using the fact that $-5x^2 + 2x + 16 = -1(5x+8)(x-2)$ from problem 1b, we have that

$$\begin{aligned} & -5x^2 + 2x + 16 = 0 \\ \Rightarrow\ & -1(5x+8)(x-2) = 0 \\ \Rightarrow\ & 5x + 8 = 0 \quad OR \quad x - 2 = 0 \text{ (Since -1 cannot equal 0)} \\ \Rightarrow\ & t = \tfrac{9}{2} \quad OR \quad t = 2 \end{aligned}$$

(c) $a^2 - 81 = 0 \quad (a = 9 \quad OR \quad a = -9)$

Using the fact that $2a^2 - 81 = (a+9)(a-9)$ from problem 1c, we have that

$$\begin{aligned} & a^2 - 81 = 0 \\ \Rightarrow\ & (a+9)(a-9) = 0 \\ \Rightarrow\ & a + 9 = 0 \quad OR \quad a - 9 = 0 \\ \Rightarrow\ & a = -9 \quad OR \quad a = 9 \end{aligned}$$

(d) $(2x^2 + 5x)(x-1)^2 = 12(x-1)^2$.

Getting all the variables onto one side, we have $(2x^2+5x)(x-1)^2 - 12(x-1)^2 = 0$. Using the fact that $(2x^2+5x)(x-1)^2 - 12(x-1)^2 = (x-1)^2(2x-3)(x+4)$ from problem 1d, we have that

$$\begin{aligned} & (2x^2 + 5x)(x-1)^2 - 12(x-1)^2 = 0 \\ \Rightarrow\ & (x-1)^2(2x-3)(x+4) = 0 \\ \Rightarrow\ & x - 1 = 0 \quad OR \quad 2x - 3 = 0 \quad OR \quad x + 4 = 0 \\ \Rightarrow\ & x = 1 \quad OR \quad x = \tfrac{3}{2} \quad OR \quad x = -4 \end{aligned}$$