Math 80
Final Exam Solutions

1. (30 pts.)

(a) (5 pts.) Find all values for which $\frac{2}{x^2-x-20}$ is undefined.

$\frac{2}{x^2-x-20}$ is undefined when $x^2-x-20=0 \Rightarrow (x-5)(x+4)=0$
$\Rightarrow x=5$ OR $x=-4$

(b) (5 pts.) Simplify the complex fraction $\frac{2+\frac{1}{x}}{\frac{6x+3}{x^2}}$.

Method 1:
$$\frac{2+\frac{1}{x}}{\frac{6x+3}{x^2}}=\frac{\frac{2x+1}{x}}{\frac{6x+3}{x^2}}=\frac{2x+1}{x}\div\frac{6x+3}{x^2}=\frac{2x+1}{x}\cdot\frac{x^2}{6x+3}=\frac{2x+1}{x}\cdot\frac{x^2}{3(2x+1)}=\frac{x}{3}$$

Method 2: The LCD of $\frac{1}{x}$ and $\frac{6x+3}{x^2}$ is x^2.
$$\frac{2+\frac{1}{x}}{\frac{6x+3}{x^2}}=\frac{x^2(2+\frac{1}{x})}{x^2(\frac{6x+3}{x^2})}=\frac{2x^2+x}{6x+3}=\frac{x(2x+1)}{3(2x+1)}=\frac{x}{3}$$

(c) (5 pts.) $\frac{x^2+3x}{2x-2}\div\frac{x+3}{2(x+2)}=$

$$\frac{x^2+3x}{2x-2}\div\frac{x+3}{2(x+2)}=\frac{x^2+3x}{2x-2}\cdot\frac{2(x+2)}{x+3}=\frac{x(x+3)}{2(x-1)}\cdot\frac{2(x+2)}{x+3}=\frac{2x(x+2)(x+3)}{2(x-1)(x+3)}$$
$$=\frac{x(x+2)}{(x-1)}$$

(d) (5 pts.) $\frac{2(x-5)^2}{x+1}\cdot\frac{x+1}{5-x}=$

$$\frac{2(x-5)^2}{x+1}\cdot\frac{x+1}{5-x}=\frac{2(x-5)(x-5)(x+1)}{(5-x)(x+1)}=\frac{-2(5-x)(x-5)(x+1)}{(5-x)(x+1)}=-2(x-5)$$

(e) (5 pts.) $\frac{2}{x}+\frac{3}{x+4}=$

The LCD is $x(x+4)$.
$$\Rightarrow \frac{2}{x}+\frac{3}{x+4}=\frac{2(x+4)}{x(x+4)}+\frac{3x}{x(x+4)}=\frac{2(x+4)+3x}{x(x+4)}=\frac{5x+8}{x(x+4)}$$

(f) (5 pts.) $\frac{2x}{x^2-6x+9}-\frac{2}{x-3}=$

The LCD is $(x-3)(x-3)$.
$$\Rightarrow \frac{2x}{x^2-6x+9}+\frac{-2}{x-3}=\frac{2x}{(x-3)(x-3)}+\frac{-2(x-3)}{(x-3)(x-3)}=\frac{2x-2(x-3)}{(x-3)(x-3)}=\frac{6}{(x-3)(x-3)}$$

2. (15 pts.) Farmer Bob has a small lettuce garden with an area of 48 square feet. The length of the garden is 3 times its width. What are the dimensions on the garden?

Let W=width of the garden. Then the length of the garden is $3W$. The area of the garden is length×width $= 3W(W) = 3W^2$. The area of the garden is also 48 square feet.

So, $3W^2 = 48 \Rightarrow 3W^2 - 48 = 0 \Rightarrow 3(W^2 - 16) = 0 \Rightarrow 3(W+4)(W-4) = 0$.

The solutions to the equation are $W = 4$ and $W = -4$. The negative solution is not valid since we are talking at the positive quantity width. So, $W = 4 \Rightarrow L = 12$

The dimensions are 4 feet × 12 feet.

3. (15 pts.) Susie Q has \$3.35 in quarters and dimes in her piggy bank. There are a total of 20 coins. How many of each coin does she have?

Let x=number of dimes in her bank. Then $20 - x$ is the number of quarters in her bank. The total dollar value in dimes is $.10x$ and the total dollar value in quarters is $.25(20 - x)$. The combined value of the quarters and dimes is \$3.35 $\Rightarrow 3.35 = .10x + .25(20-x) = .10x + 5 - .25x = -.15x + 5 \Rightarrow -1.35 = -.15x \Rightarrow x = 9$

So there are 9 dimes and 11 quarters in the bank.

4. (20 pts.)

(a) (10 pts.) Find an equation for the line through the points $(-3, 4)$ and $(1, \frac{8}{3})$. Write your answer in slope-intercept form.

Slope$=\dfrac{4-\frac{8}{3}}{-3-1}=\dfrac{\frac{12}{3}-\frac{8}{3}}{-3-1}=\dfrac{\frac{4}{3}}{-3-1}=\dfrac{4}{3}\cdot\dfrac{-1}{4}=\dfrac{-1}{3}$

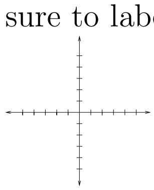
Using the point-slope formula with slope$=\frac{-1}{3}$ and the point $(-3, 4)$, $y - 4 = \frac{-1}{3}(x+3)$

Solving for the variable y and simplifying, $y = \frac{-1}{3}(x+3)+4 = \frac{-1}{3}x - 1 + 4 = \frac{-1}{3}x + 3$
So $y = \frac{-1}{3}x + 3$

(b) (5 pts.) Find an equation for a line through the point $(0, -2)$ that is perpendicular to the line in part a. Write your answer in slope-intercept form.

The slope of the line perpendicular the line in part a has slope 3 (negative reciprocal) and the y-intercept $(0, -2)$. So the equation of the line using slope-intercept form is $y = 3x - 2$.

(c) (5 pts.) Graph the lines for the equations in part a and b on the axis below. Be sure to label each graph.



5. (15 pts.) Solve the following equations.

(a) (5 pts.) $2p + 5(p-1) = 4p + 1$

$$2p + 5(p-1) = 4p + 1 \;\Rightarrow\; 2p + 5p - 5 = 4p + 1 \;\Rightarrow\; 7p - 5 = 4p + 1$$

$$\Rightarrow\; 3p = 6 \;\Rightarrow\; p = 2$$

(b) (5 pts.) $3r^2 = 8r - 5$

$$3r^2 = 8r - 5 \;\Rightarrow\; 3r^2 - 8r + 5 = 0 \;\Rightarrow\; (3r-5)(r-1) = 0 \;\Rightarrow\; r = 5/3 \text{ OR } r = 1$$

(c) (5 pts.) $4x^3 - 16x = 0$

$$4x^3 - 16x = 0 \;\Rightarrow\; 4x(x^2 - 4) = 0 \;\Rightarrow\; 4x(x+2)(x-2) = 0 \;\Rightarrow\; x = 0$$
$$\text{OR } x = 2 \text{ OR } x = -2$$

6. (20 pts.) Carry out the following operations.

(a) (5 pts.) $(2x+5)\cdot(3y - x + 2) =$

$$\begin{aligned}(2x+5)\cdot(3y-x+2) = 2x(3y-x+2)+5(3y-x+2) &= 6xy-2x^2+4x+15y-5x+10\\ &= 6xy - 2x^2 - x + 15y + 10\end{aligned}$$

(b) (5 pts.) Simplify $\frac{(3a^2b^{-4})(2ab)}{(5a^3b)^{-1}}$. Write your answer with only positive exponents.

$$\frac{(3a^2b^{-4})(2ab)}{(5a^3b)^{-1}} = \frac{6a^3b^{-3}}{(5^{-1}a^{-3}b^{-1})} = 6(5)a^6b^{-2} = \frac{30a^6}{b^2}$$

(c) (5 pts.) $\frac{3xy - 24x^2y + 9y^2}{3xy} =$

$$\frac{3xy-24x^2y+9y^2}{3xy} = \frac{3xy}{3xy} - \frac{24x^2y}{3xy} + \frac{9y^2}{3xy} = 1 - 8x + \frac{3y}{x}$$

(d) (5 pts.) $\left(\frac{1}{9}\right)\cdot\left(\frac{2}{3}\right)^{-3} =$

$$\left(\tfrac{1}{9}\right)\cdot\left(\tfrac{2}{3}\right)^{-3} = \left(\tfrac{1}{9}\right)\cdot\left(\tfrac{3}{2}\right)^{3} = \left(\tfrac{1}{9}\right)\cdot\left(\tfrac{27}{8}\right) = \tfrac{3}{8}$$