

Math 80
Midterm 2 Solutions

NOTE: Particular methods of solution may vary. For example, exponent rules can be applied in a different order and still yield the same answer.

1. (10 pts.) Find an equation for the line through $(2, -1)$ with slope $\frac{1}{2}$. Write the equation in slope-intercept form and graph on the axis below.

Using the point-slope formula we have that the equation for the line is $y - (-1) = \frac{1}{2}(x - 2)$ or $y + 1 = \frac{1}{2}(x - 2)$. Solving for y , we have $y = \frac{1}{2}(x - 2) - 1 = \frac{1}{2}x - 1 - 1 = \frac{1}{2}x - 2$, which is the slope-intercept form of the equation.

To graph, draw the point $(2, -1)$ and use the slope $\frac{1}{2}$ to draw the next point or find another point given the equation of the line and draw the line through the points.

2. (10 pts.) The average distance from Mars to the sun is 2.3×10^8 kilometers. The speed of light is approximately 3.0×10^5 kilometers per second. On average, how long in minutes does it take for sun light to reach Mars' surface? Make sure to do your calculations using scientific notation, but round your answer to the nearest whole minute.

Using the $d = rt$ equation, we have the $t = \frac{d}{r}$ where d is the distance and r is the speed of light. So, the time it takes sun light to reach Mars is

$$\begin{aligned} t &= \frac{2.3 \times 10^8 \text{ kilometers}}{3.0 \times 10^5 \text{ kilometers/second}} = \frac{2.3}{3.0} \times \frac{10^8}{10^5} \approx .7667 \times 10^3 = 7.667 \times 10^2 \\ &= 766.7 \text{ seconds} = \frac{766.7}{60} \text{ minutes} \\ &\approx 12.78 \text{ minutes.} \end{aligned}$$

So it takes about 13 minutes for sun light to reach Mars' surface.

3. (15 pts.) Simplify the following exponential expressions **as much as possible**. Write answers with only positive exponents.

(a) (5 pts.) $\left(-\frac{1}{2}\right)^{-4}$

$$\left(-\frac{1}{2}\right)^{-4} = \frac{1}{\left(-\frac{1}{2}\right)^4} = \frac{1}{\frac{1}{16}} = 16$$

(b) (5 pts.) $(5x^3y^2z)^3 \cdot (yz^2)^4$

$$\begin{aligned} (5x^3y^2z)^3 \cdot (yz^2)^4 &= (5^3x^{3 \cdot 3}y^{2 \cdot 3}z^3) \cdot (y^4z^{2 \cdot 4}) = (125x^9y^6z^3) \cdot (y^4z^8) \\ &= 125x^9y^{6+4}z^{3+8} = 125x^9y^{10}z^{11} \end{aligned}$$

(c) (5 pts.) $\frac{(m^{-6}n)^{-2}}{m^{-1}n} + m^2n$

$$\begin{aligned} \frac{(m^{-6}n)^{-2}}{m^{-1}n} + m^2n &= \frac{m^{-6 \cdot (-2)}n^{-2}}{m^{-1}n} + m^2n = \frac{m^{12}n^{-2}}{m^{-1}n} + m^2n = m^{12 - (-1)}n^{-2-1} + m^2n \\ &= m^{13}n^{-3} + m^2n = \frac{m^{13}}{n^3} + m^2n \end{aligned}$$

4. (25 pts.) Carry out the following operations and simplify as much as possible.

(a) (5 pts.) $(-2x^6 + 3x^4 - x^2) - (x^6 + 2x^4 + 3x^2)$
 $(-2x^6 + 3x^4 - x^2) - (x^6 + 2x^4 + 3x^2) = (-2x^6 + 3x^4 - x^2) + (-x^6 - 2x^4 - 3x^2)$
 $= -3x^6 + x^4 - 4x^2$

(b) (5 pts.) $(3m + 5) \cdot (m - 2)$

$$\text{FOILING} \Rightarrow (3m + 5) \cdot (m - 2) = 3m^2 - 6m + 5m - 10 = 3m^2 - m - 10$$

(c) (8 pts.) $(x - 1) \cdot (2xy^2 - 3x + y^4)$

$$\begin{aligned} \text{Distributing} \Rightarrow (x - 1) \cdot (2xy^2 - 3x + y^4) &= x \cdot (2xy^2 - 3x + y^4) - 1 \cdot (2xy^2 - 3x + y^4) \\ &= 2x^2y^2 - 3x^2 + xy^4 - 2xy^2 + 3x - y^4 \end{aligned}$$

(d) (7 pts.) $\frac{2x^3y + 6x^2y^3 - 2x}{2x}$

$$\frac{2x^3y + 6x^2y^3 - 2x}{2x} = \frac{2x^3y}{2x} + \frac{6x^2y^3}{2x} - \frac{2x}{2x} = x^2y + 3xy^3 - 1$$

5. (15 pts.) Factor the following polynomials as much as possible.

(a) (5 pts.) $2x^2 + 6x - 8$

$$2x^2 + 6x - 8 = 2(x^2 + 3x - 4) = 2(x + 4)(x - 1)$$

(b) (5 pts.) $12a^2b + 4a^3b^2 - 8a^4b^2$

$$12a^2b + 4a^3b^2 - 8a^4b^2 = 4a^2b(3 + ab - 2a^2b)$$

(c) (5 pts.) $6m^2 + mn - 24m - 4n$ (Factor by grouping.)

$$\begin{aligned} 6m^2 + mn - 24m - 4n &= m(6m) + m(n) - 4(6m) - 4(n) \\ &= m(6m + n) - 4(6m + n) = (m - 4)(6m + n) \end{aligned}$$