

Math 207 Quiz #8 Answers

1. (a) Using the formula: $l = \frac{mg}{k} \Rightarrow 1.635 = \frac{4(9.81)}{k} \Rightarrow k = \boxed{24 \text{ N/m}}$

(b) Using the formula (or matching coefficients):

$$y_p = \frac{6}{\sqrt{(24-4(2^2))^2 + 3^2(2^2)}} \sin(3t + \phi) \Rightarrow \boxed{y_p = \frac{3}{5} \sin(3t + \phi) \quad \text{where } \phi = \arctan\left(\frac{4}{3}\right)}$$

$$\text{Max displacement} = \boxed{\frac{3}{5} \text{ meters}}$$

(c) $\gamma_r = \sqrt{\frac{24}{4} - \frac{3^2}{2(4^2)}} = \sqrt{\frac{183}{32}}$

$$\text{Resonance Frequency: } \boxed{\frac{\sqrt{\frac{183}{32}}}{2\pi} \approx 0.3806 \text{ cycles/sec}}$$

(d) If the system was at resonance, then the external force would be $F(t) = 6 \cos\left(\sqrt{\frac{183}{32}}t\right)$ and the coefficient on the steady-state solution, which gives the maximum displacement, would be

$$\frac{6}{\sqrt{(24-4\left(\sqrt{\frac{183}{32}}\right)^2)^2 + 3^2\left(\sqrt{\frac{183}{32}}\right)^2}} = \boxed{\frac{48}{5\sqrt{135}} \approx 0.8262 \text{ meters.}}$$