Math 207 Quiz #6 Answers

1. Auxiliary Equation: $r^2 - 4r + 13 = 0 \quad \Rightarrow \quad r = \frac{4 \pm \sqrt{-36}}{2} = 2 \pm 3i$

   General Solution: $\boxed{y = c_1 e^{2t} \cos 3t + c_2 e^{2t} \sin 3t}$

2. Auxiliary Equation: $4r^2 + 13r + 3 = (4r + 1)(r + 3) = 0 \quad \Rightarrow \quad r = -\frac{1}{4}, \quad r = -3$

   General Solution: $y = c_1 e^{-t/4} + c_2 e^{-3t}$

   The initial conditions yield the following equations: $c_1 + c_2 = 2, \quad -\frac{1}{4} c_1 - 3c_2 = 5$

   The solution is $c_1 = 4, \quad c_2 = -2$

   Solution: $\boxed{y = 4e^{-t/4} - 2e^{-3t}}$

3. In order to have a solution that is overdamped, we need $b^2 - 4mk > 0 \quad \Rightarrow \quad 25 - 4(3)k > 0 \quad \Rightarrow \quad \boxed{k < \frac{25}{12}}$

4. The particular solution will have the form $y_p = At + B$. Plugging this into the equation, and matching the coefficients will yield the equations: $-A = 5, \quad \text{and} \quad 2A - B = -3$

   The solution is $A = -5, \quad B = -7. \quad \Rightarrow \quad \boxed{y_p = -5t - 7}$