

## Math 207 Quiz #4 Answers

1. This equation is a Bernoulli equation. Dividing by  $y^2$ :  $y^{-2} \frac{dy}{dx} - \frac{4y^{-1}}{x} = \frac{1}{x^3}$

Substitute:  $v = y^{-1} \quad \frac{dv}{dx} = -y^{-2} \frac{dy}{dx}$

The equation now becomes  $-\frac{dv}{dx} - \frac{4v}{x} = \frac{1}{x^3} \Rightarrow \frac{dv}{dx} + \frac{4v}{x} = -\frac{1}{x^3}$

Now the equation is linear with an integrating factor of  $\mu(x) = x^4$ .

Solution to the linear equation:  $x^4 v = -\frac{1}{2}x^2 + C$  or  $v = -\frac{1}{2}x^{-2} + Cx^{-4}$

Back to  $y$ :  $y^{-1} = -\frac{1}{2}x^{-2} + Cx^{-4}$  OR  $y = \frac{1}{-\frac{1}{2}x^{-2} + Cx^{-4}}$

Note:  $y \equiv 0$  is a lost solution.

2. Let  $B$  = Amount of bleach (in liters) in the container at  $t$  minutes

Then  $\frac{dB}{dt} = 0.01(4) - 4\left(\frac{B}{50}\right)$ , with  $B(0) = 0.1(50) = 5$  liters

This is a linear or a separable equation with a solution of  $B = 0.5 + Ce^{-0.08t}$

Using the initial condition:  $C = 4.5$

So, to answer the question, solve  $2.5 = 0.5 + 4.5e^{-0.08t} \Rightarrow t = -\frac{1}{0.08} \ln\left(\frac{2}{4.5}\right) \approx \boxed{10.14 \text{ minutes}}$