Math 207 Quiz #3 Solutions

1. (a) Note that $\frac{\partial M}{\partial y} = 1$ and $\frac{\partial N}{\partial x} = 2xy - 1$. So this is not exact, but since

$\frac{\partial M/\partial y - \partial N/\partial x}{N} = \frac{2-2xy}{x^2y-x} = \frac{-2(xy-1)}{x(xy-1)} = \frac{-2}{x}$ depends only on x, we can multiply by the integrating factor $\mu(x) = x^{-2}$.

New **exact** equation: $(2 + yx^{-2})dx + (y - x^{-1})dy = 0$

Finding $F(x, y)$: $F(x, y) = \int 2 + yx^{-2}\, dx \ = \ 2x - yx^{-1} + g(y)$ for some function g

To find $g(y)$: $\frac{\partial}{\partial y}[2x - yx^{-1} + g(y)] = y - x^{-1} \quad \leftarrow N$

$$-x^{-1} + g'(y) = y - x^{-1} \quad \Rightarrow \quad g'(y) = y \quad \Rightarrow \quad g(y) = \frac{y^2}{2}$$

Solutions: $\boxed{2x - yx^{-1} + \frac{y^2}{2} = \mathrm{C}}$

(b) Substituting $v = \frac{y}{x} \quad \Rightarrow \quad y = vx,$ and $\frac{dy}{dx} = x\frac{dv}{dx} + v$:

$$x\frac{dv}{dx} + v = \frac{x \sec v + vx}{x} \quad \Rightarrow \quad x\frac{dv}{dx} = \sec v$$

Solving this new separable equation yields: $\boxed{y = x \arcsin(\ln|x| + C)}$