Math 207 Quiz #2 Answers

1. (a) This equation is linear. In standard form, it is $\frac{dy}{dx} + \frac{3}{x}y = \frac{e^x}{x^2}$.

Multiplying through by an integrating factor is $\mu(x) = x^3$:

$$x^3\frac{dy}{dx} + 3x^2y = xe^x$$
$$\frac{d}{dx}[x^3y] = xe^x$$
$$x^3y = \int xe^x\,dx$$

Using integration by parts with $u = x$ and $dv = e^x\,dx$:

$$\boxed{x^3y = xe^x - e^x + C}$$

OR $\boxed{y = \frac{xe^x - e^x + C}{x^3}}$

(b) This equation is separable. You can rewrite it as $\frac{dy}{dt} = t^3(t^4+1)\cos^2 y$

OR $\sec^2 y\,dy = t^3(t^4+1)\,dt$

Integrating both sides: $\int \sec^2 y\,dy = \int t^7 + t^3\,dt$

$\tan y = \frac{1}{8}t^8 + \frac{1}{4}t^4 + C$ (Using substitution to integrate the right side)

Plugging in $t = 1$ and $y = \pi$: $\tan\pi = \frac{1}{8} + \frac{1}{4} + C \quad \Rightarrow \quad C = -\frac{3}{8}$

Solution: $\boxed{\tan y = \frac{1}{8}t^8 + \frac{1}{4}t^4 - \frac{3}{8}}$