Math 207 Quiz #1 Answers

1. (a) For $y = t^2 + C_1e^{3t} + C_2e^{-3t}$, we have that $\frac{d^2y}{dt^2} = 2 + 9C_1e^{3t} + 9C_2e^{-3t}$.

   Since $2 + 9C_1e^{3t} + 9C_2e^{-3t} + 9t^2 = 9(t^2 + C_1e^{3t} + C_2e^{-3t}) + 2$, we can see that the given function is a general solution.

   (b) Given that $y(0) = 6$ and that the general solution is $y = t^2 + C_1e^{3t} + C_2e^{-3t}$, we have that $6 = C_1 + C_2$.

   Given that $y'(0) = -6$ and that $y' = 2t + 3C_1e^{3t} - 3C_2e^{-3t}$, we have that $-6 = 3C_1 - 3C_2$.

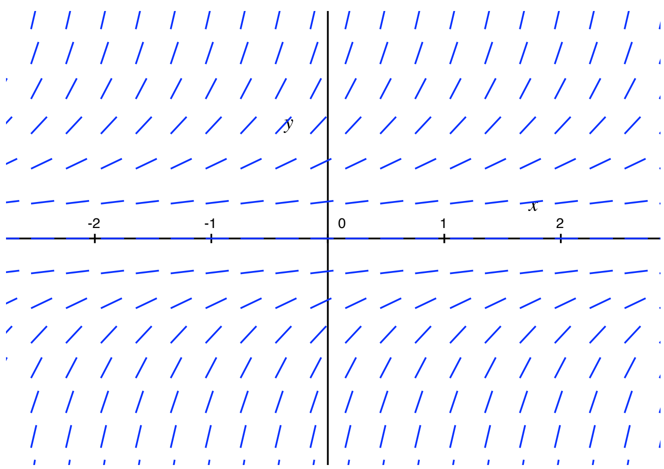
   Solving the system: $C_1 = 2, \quad C_2 = 4$ So, the solution is $\boxed{y = t^2 + 2e^{3t} + 4e^{-3t}.}$

2. Implicit Differentiation: $\frac{d}{dx}[x^2y + 3y^4] = \frac{d}{dx}[\sin(x)] \quad \Rightarrow \quad 2xy + x^2y' + 12y^3y' = \cos(x)$

   Solving for y': $y' = \frac{\cos(x) - 2xy}{x^2 + 12y^3}$

   So $\boxed{\text{yes,}}$ it is a solution.

3. Note that the isoclines for this direction field will be horizontal lines. You should have a direction field that has positive slopes for $y \neq 0$. The further the y-value is away from the axis, the steeper the slope. When $y = 0$, the slope is 0.



   So, as $x \to \infty$, we have that the $\boxed{\text{solutions above the } x\text{-axis go to infinity,}}$ and that $\boxed{\text{solutions below the } x\text{-axis go to 0.}}$

   (Note: Solutions cannot cross the equilibrium solution $y = 0$, due to the uniqueness of solutions (See theorem 1 in §1.2)

4. Step 0: $x_0 = 0, \quad y_0 = 5$

   Step 1: $x_1 = 1, \quad y_1 = 5 + 1f(0,5)$
   $= 5 + 3(0) - 5$
   $= 0$

Step 2: $x_2 = 2, \quad y_2 = 0 + 1f(1,0)$

$$\begin{aligned} &= 3(1) - 0 \\ &= 3 \end{aligned}$$

So, $\boxed{y(2) \approx 3}$