

## Differential Equations - Practice Problems for the Last Three Weeks of Material

These practice problems are a collection from other exams from other instructors. They are good practice, but do not encompass all of the material from the last three weeks. Make sure to do homework #10!

1. A 1 kg-mass is attached to a spring with stiffness 4 N/m. There is no damping on the system. The mass is pulled 50 centimeters to the right of equilibrium and given an initial leftward velocity of 4 m/sec.
  - (a) Find the amplitude and frequency of the resulting displacement function.
  - (b) When will the mass first pass through the equilibrium position?
  - (c) Suppose the same system is hung vertically. Gravity pulls the mass to a new rest position. How far will the spring stretch to get to its new equilibrium?
  - (d) What is the resonance frequency of the system?
2. A 1 kg-mass is attached to a spring with stiffness 4 N/m. The damping constant for the system is 5 N-sec/m.
  - (a) If the mass is pushed rightward from the equilibrium position with a velocity of 1 m/sec, when will it attain its maximum displacement to the right?
  - (b) An external force of  $F(t) = 2 \cos(3t)$  is applied to the system. Find the steady-state solution for the system.
3. Find the general solution of the following system 
$$\begin{cases} x' - 3x - 2y = 2e^{-t} \\ y' + 2x + y = e^{-t} \end{cases}$$
4. Find the solution to the system from #3 that has initial conditions  $x(0) = 5, y(0) = -2$ .
5. A building consists of two rooms A and B. Room A has an air conditioner that cools the room down by  $7^\circ\text{F}$  each hour. The time constant between room A and the outside is 4 hours. The time constant between room B and the outside is 1 hour. The time constant between the two rooms is 2 hours. If the outside temperature stays constant at  $100^\circ$ , how warm will it eventually get in room B?
6. Consider the system of differential equations
$$\frac{dx}{dt} = \frac{4}{y}$$
$$\frac{dy}{dt} = \frac{2}{x}.$$
  - (a) Solve the phase plane equation. (You can write your answer explicitly.)
  - (b) Sketch several representative trajectories (with their flow arrows).