Math 207
Final Exam
June 10th, 2011

Name: ________________________________

1. Your exam contains 6 questions and 9 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 1 hour and 50 minutes for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. Leave answers in exact form (as simplified as possible).
5. Put a box around your final answer where applicable.
6. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic). You are allowed one 8.5”×11” notesheet (both sides).
7. If you need extra space, attach a sheet to the back of the exam and clearly indicate this.
8. Note: Do not be intimidated by the amount of space provided! I wanted to ensure that you had more than enough space for each problem. This does not mean that I expect you to fill the space.

Problem	Total Points	Score
1	25	
2	18	
3	14	
4	20	
5	14	
6	12	
Total	100	

1. (25 pts.) Find the general solution to each equation below.

(a) (8 pts.) $x^2 \cdot \dfrac{dy}{dx} + 4xy = 9x^3 + 3$ (Write your answer **explicitly**.)

(b) (8 pts.) $\dfrac{dy}{dx} - x^2 y^2 \sec^2(x^3 + 1) = 0$ (Write your answer **explicitly**.)

#1 Continued on the Next Page →

#1 Continued:

(c) (9 pts.) $2x^7y\ dx + (x^8 + 15y^{-1})dy = 0$ (You may write your answer **implicitly**.)

2. (18 pts.) Suppose you enter a shed that is initially 85°F. The temperature outside the shed remains constant at 100°F. You are going to work in the shed for many hours so you turn on the air conditioner when you enter. The air conditioner has the capacity to cool the shed by 8°F per hour. The time constant for the shed is 2 hours.

   (a) (15 pts.) Find a formula for the temperature T (in °F) of the shed t hours after you enter.

   (b) (3 pts.) According to this model, does the internal temperature of the shed ever reach 90°F? If so, when?

3. (14 pts.) Find a general solution of the system $\begin{cases} x' &= 3x - y \\ y' &= 4x - y + t \end{cases}$ with either substitution or elimination.

(Note: All differentiation is with respect to t. You should have only 2 unkonwn constants in your solutions. Simplify your answers as much as possible.)

4. (17 pts.) A 1-kg mass is attached to a spring with a spring constant of 25 N/m. The damping constant for the system is 6 N-sec/m. Assume there are no external forces on the system. Yay!
At time $t = 0$, the mass is displaced 8 meters from the equilibrium position (in the positive direction) and released.

(a) (13 pts.) Determine the equation of motion of the mass.

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#4 Continued:

(b) (4 pts.) Find the first time ($t > 0$) at which the mass passes through the equilibrium position.

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5. (14 pts.) Consider the system of differential equations $\begin{cases} \dfrac{dx}{dt} = x - 1 \\ \dfrac{dy}{dt} = 2y \end{cases}$

(a) (2 pts.) Determine the set of equilibrium solutions for the system.

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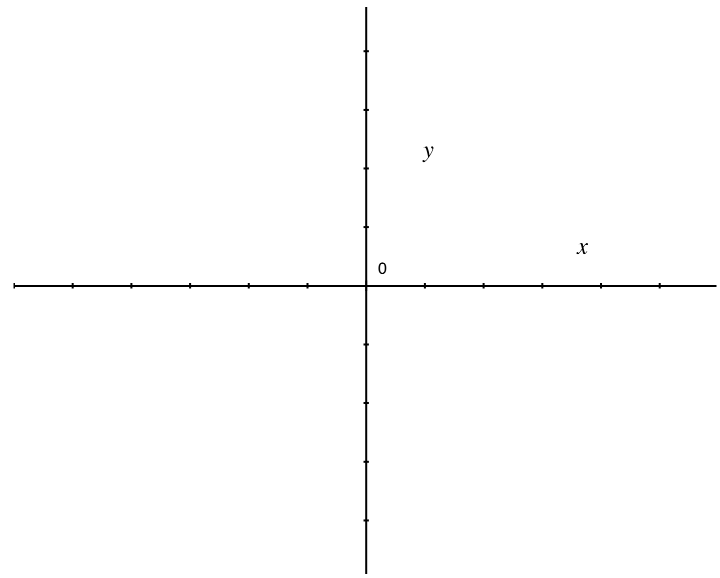
#5 Continued:

(b) (10 pts.) Solve the phase-plane equation for the system $\begin{cases} \dfrac{dx}{dt} &= x - 1 \\ \dfrac{dy}{dt} &= 2y \end{cases}$. Then sketch several representative trajectories (with their flow arrows). Write the solution **explicitly**.



(c) (2 pts.) Consider the solution of the above system with the initial values of $x(0) = -2$, $y(0) = 4$. Given what you see in the phase-plane, what happens to $x(t)$ and $y(t)$ as $t \to \infty$?

6. (16 pts.) Two tanks, each initially containing 40 liters of pure water, are interconnected by pipes with liquid flowing from tank A into tank B at a rate of 5 L/min and from tank B to tank A at a rate of 2 L/min. Liquid flows out of tank B at a rate of 3 L/min. A brine solution with a concentration of 0.2 kg of salt/L flows into tank A at a rate of 3 L/min. Assume the tanks are well-stirred.

   (a) (10 pts.) Set up, but **do not solve**, a system of differential equations modeling changes to the amount of salt in each tank. Be sure to **define any variables** you use and give **initial conditions**.

   (b) (2 pts.) What will happen to the concentration of salt in each tank as time goes on? (You should be able to answer this without solving the system.)