Differential Equations - Exam #2 Practice Problems

These practice problems are a collection from other exams from other instructors.

1. A tank with a capacity of 500 gallons originally contains 200 gallons of water with 100 pounds of salt. Water containing 1 lb of salt/gallon is pumped into the tank at a rate of 3 gallons/minute.

   (a) Write an initial value problem modeling the situation if the well-mixed solution flows out of the tank at 3 gallons/minute. **Do not solve the equation.** Define all variables you use.

   (b) Find all equilibrium solutions to the DE in part (a) and explain the significance of the equilibrium solutions in terms of the amount of salt in the tank.

   (c) Suppose instead that the well-mixed solution flows out of the tank at a rate of 2 gallons/minute. Determine the concentration of salt (in lbs/gallon) when the tank is at the point of overflowing.

2. A car is running with the air conditioner on with a constant internal temperature of 70°F. The car is turned off when the driver parks into a garage. The temperature in the garage is a constant 100°F for the next several hours. If the time constant of the car is 3/4 of an hour, when will the car reach 95°F? Define all variables you use.

3. An object of mass 100 kg is released from rest from a boat into the water and allowed to sink. While the force of gravity is pulling the object down, water resistance exerts a force on the object that is directly proportional to the velocity of the object with a proportionality constant of 10 N-sec/m.

   (a) Write an initial value problem that models the situation. Define all variables you use. **You do not need to solve the equation in this part.**

   (b) Find the equilibrium solution to the DE in part (a). What is the significance of the equilibrium solution in terms of the motion of the object?

   (c) Find the position of the object at a time t.

4. Solve the initial value problem $2y'' + y' + \frac{5}{4}y = 2e^{-t}, \quad y(0) = 0,\ y'(0) = 1.$

5. Determine a suitable form for a particular solution y_p for $2y'' + 3y' + y \ = \ t^2 + 3\cos t + te^{-t}$.

6. Find a general solution to $y'' - 6y' + 9y = 6t^2 - 12e^{3t}$.

7. A 1 kg mass is attached to a spring with a spring constant of 5 N/m. The damping constant is 4 N-sec/m. If there is an external driving force of $4\cos(5t)$ Newtons, find the equation of motion of the mass if the initial position at time 0 is 1 meter to the right and the initial velocity is 0 m/s.

8. Give a specific example of an equation that satisfies the given condition. You do not need to solve your equation. You do not need to describe your equation.

   (a) A linear second-order differential equation with constant coefficients that models an underdamped mass-spring system.

   (b) A linear second-order homogeneous differential equation with constant coefficients whose general solution is $y = c_1 e^{4t} + c_2 e^{-2t}$.

   (c) A linear second-order differential equation with constant coefficients that has a particular solution of the form $y_p = t(At + B)e^{-7t}$.

   (d) A linear second-order differential equation with constant coefficients that models an overdamped mass-spring system.