Math 207
Exam 2 Answers

1. Let $A(t)$ = Amount of acid in the tank (in gallons) at t minutes

   IVP: $\dfrac{dA}{dt} = 4(0.1) - 2(\frac{A}{200+2t}) = 0.4 - \frac{A}{100+t}$, $\quad A(0) = 0.1(200) = 2$ gallons

   This is a linear equation with solution $\boxed{A = 0.2(100+t) - \frac{1800}{100+t}}$

   OR $\boxed{A = \frac{0.2t^2+40t+200}{100+t}}$

2. (a) IVP: $\boxed{\dfrac{dP}{dt} = 2P - 400,000, \quad P(0) = 150,000}$

   Note that with the initial condition $\dfrac{dP}{dt}$ is negative, which means that the population is decreasing. In fact, if $P < 200,000$, $\dfrac{dP}{dt}$ is negative. So, the population will continue to decrease at a faster and faster rate at time goes on. $\boxed{\text{Eventually, the population will reach 0.}}$ (Note: You could also solve the diff. eq to see this or look at a direction field.)

   (b) If A is the fishing rate in fish/year, then $\dfrac{dP}{dt} = 2P - A$. In order for the cod population to remain constant at 150,000, we need $\dfrac{dP}{dt} = 2(150,000) - A = 0 \quad \Rightarrow \quad A = \boxed{\text{300,000 fish/year}}$

3. (a) IVP: $2y'' + 4y' + 24y = 0, \quad y(0) = -2, \quad y'(0) = 14$

   Solutions to the Aux. Equation: $r = -1 \pm 4i$

   Solution: $y = c_1 e^{-t}\cos(4t) + c_2 e^{-t}\sin(4t)$

   Using initial conditions: $c_1 = -2, \quad c_2 = 3 \quad \Rightarrow \quad \boxed{y = -2e^{-t}\cos(4t) + 3e^{-t}\sin(4t)}$

   (b) The object will oscillate around the equilibrium as $t \to \infty$. The amplitude of the oscillation will approach 0.

4. Solutions to the Aux. Equation: $r = 0, \; r = 2$

   Homogeneous Solution: $y_h = c_1 + c_2 e^{2t}$

   Particular Solution Form: $y_p = A\cos t + B\sin t$

Matching Coefficients to find A & B: $A = -\frac{1}{5}$, $B = -\frac{2}{5}$

Solution: $\boxed{y = c_1 + c_2 e^{-2t} - \frac{1}{5}\cos t - \frac{2}{5}\sin t}$

5. Answers may vary. Here's one solution.

Since the general solution ot a linear second-order differential equation with constant coefficients can consist of a homogeneous solution and a particular solution, we can look at the terms with the constants as the homogeneous solution and the rest as the particular solution. $y_h = c_1 e^{3t} + c_2 t e^{3t}$, $y_p = 2t^2 + 1$.

The homogeneous solution poiints to an auxiliary equation with a double root of $r = 3$. $(r-3)^2 = r^2 - 6r + 9 = 0$ is such an equation.

If this is the auxiliary equation, then the differential equation must look like $y'' - 6y' + 9y = f(t)$.

To find $f(t)$, keep in mind that the particular solution must yield $f(t)$ when it is plugged into $y'' - 6y' + 9y$. Since $y_p'' - 6y_p + 9y_p = 18t^2 - 24t + 13$, we must have that $f(t) = 18t^2 - 24t + 13$.

Differential EQ: $\boxed{y'' - 6y' + 9y = 18t^2 - 24t + 13}$