

Math 207
Exam 2
May 20th, 2011

Name: _____

1. Your exam contains 5 questions and 6 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 50 minutes for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. Leave answers in exact form (as simplified as possible).
5. Put a box around your final answer where applicable.
6. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic). You are allowed one 4"×6" notecard (both sides).
7. If you need extra space, attach a sheet to the back of the exam and clearly indicate this.
8. Note: Do not be intimidated by the amount of space provided! I wanted to ensure that you had more than enough space for each problem. This does not mean that I expect you to fill the space.
9. **Define all variables you use** when writing differential equations or initial value problems.

Problem	Total Points	Score
1	22	
2	20	
3	23	
4	20	
5	15	
Total	100	

1. (22 pts.) A large tank initially contains 200 gallons of a solution that is 1% acid. A 10% acid solution is pumped into the tank at a rate of 4 gal/min, and the well-stirred mixture flows out of the tank at a rate of 2 gal/min.

Find a formula for the amount of acid in the tank (in gallons) at t minutes.

2. (20 pts.) The population of cod in a certain area is initially 150,000. If left alone, the population would increase at a rate **proportional** to the population, with a proportionality constant of $2/\text{year}$. However, commercial fishing in the area removes fish at a constant rate of 400,000 fish/year.

(a) (13 pts.) Write an initial value problem modeling the rate of change of the cod population P over time t . Use your model to predict what will eventually happen to the cod population as $t \rightarrow \infty$. **You do not need to solve the DE to answer this question.**

(b) (7 pts.) In order for the cod population to remain **constant** at 150,000, what would the commercial fishing rate have to be? **You do not need to solve a DE to answer this question.**

3. (23 pts.) A 2-kg mass is attached to a spring with stiffness 34 N/m. The damping constant for the system is 4 N-sec/m. There are no external forces acting on the system. Initially, the mass is 2 units to the left of equilibrium and the velocity is 14 m/s to the right.

(a) (17 pts.) Find the equation of motion of the object.

(b) (6 pts.) **Briefly** describe the motion of the mass at $t \rightarrow \infty$.

4. (20 pts.) Find the general solution of $y'' - 2y' = \cos t$.

5. (15 pts.) Find a linear second-order differential equation with constant coefficients whose **general** solution is given by

$$y = c_1 e^{3t} + c_2 t e^{3t} + 2t^2 + 1,$$

where c_1 and c_2 are arbitrary constants.