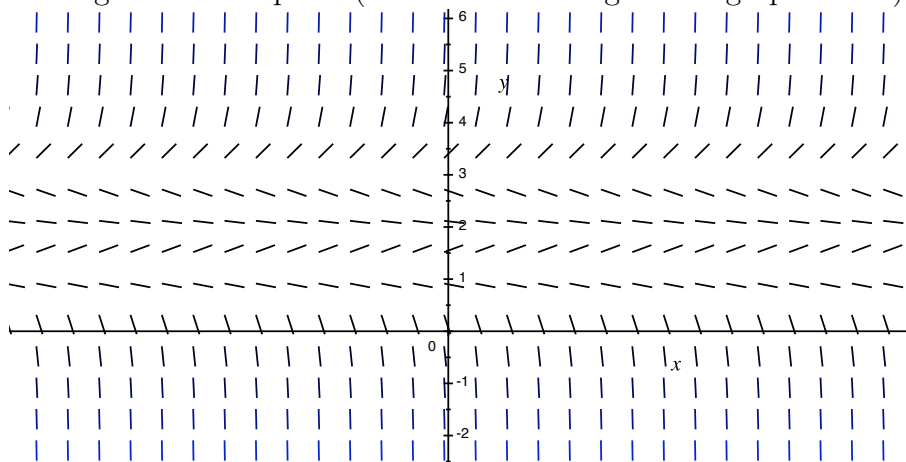


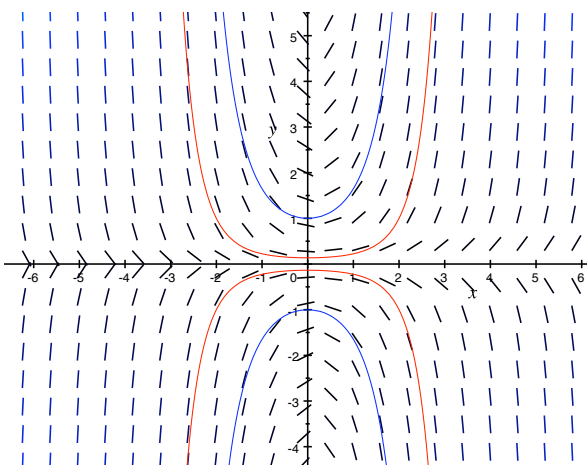
Differential Equations - Exam #1 Practice Problems Answers

1. Since both $f(x, y) = y^{1/3}$ and $\frac{\partial f}{\partial y} = \frac{1}{3}y^{-2/3}$ is continuous when $x = 0$ and $y = 2$, there is a unique solution to the IVP defined on some interval of x around $x = 0$.
2. Since $\frac{\partial f}{\partial y} = \frac{1}{3}y^{-2/3}$ is not continuous when $x = 0$ and $y = 0$, we cannot conclude that there is a unique solution to the IVP defined on some interval of x around $x = 0$.
3. Since $\frac{dy}{dx} = (y - 2)(y - 1)(y - 3)$, the equilibrium solutions are $y \equiv 3$, $y \equiv 2$, $y \equiv 1$. Here is the direction field. Note: Along the isoclines $y = 1$, $y = 2$, $y = 3$, there should be line segments of slope 0. (I cannot seem to get it to graph these.)



Solutions with initial y -values less than 1 should approach $-\infty$ as $t \rightarrow \infty$. Solutions with initial y -values between 1 and 2 should approach 2 as $t \rightarrow \infty$. Solutions with initial y -values between 2 and 3 should approach 2 as $t \rightarrow \infty$. Solutions with initial y -values greater than 3 should approach ∞ as $t \rightarrow \infty$.

4. Using isoclines, $\frac{dy}{dx} = C$ when $xy = C$
 $\Rightarrow y = \frac{C}{x}$. This yields the following direction field with some sketched solutions.



5. Using Euler's method with 3 steps: $y(0.3) \approx 0.5497$
6. (a) This equation is separable. The solution is $y = \sqrt{e^{x^3/3 + \sin x} - \frac{1}{2}}$

- (b) This is a linear equation, which is more evident once you multiply both sides by r . The integrating factor is $\mu(\theta) = \sec \theta$. The solution is $r = \sin \theta + 2 \cos \theta$.
- (c) This equation is exact. The solutions are $xy^2 - x^2 + y = C$.
- (d) This equation can be solved using special integrating factors to make it an exact equation. The integrating factor is $\mu(y) = y^{-3}$. The solutions are $x^2 + xy + y^{-2} = C$ and $y \equiv 0$.
- (e) This equation is homogeneous. The solutions are $(\frac{y}{x} + 1)^{-2}(\frac{y}{x} + 2)^3 = Cx^{-1}$ or $(y + 2x)^3 = C(y + x)^2$.
- (f) This is a linear equation. The integrating factor is $\mu(t) = e^{\sin t}$. The solutions are $y = te^{-\sin t} + 1 + Ce^{-\sin t}$.
- (g) This equation is homogeneous or you can use special integrating factors. The solutions are $y^2 = -2x^2 \ln|x| + Cx^2$.
- (h) This equation is separable. The solution is $\frac{y^2}{2} - y = -\cos x + 1.5e^x + 2$ or $y = 1 - \sqrt{-2\cos x + 3e^x + 3}$.
- (i) This equation is linear or it can be solved using special integrating factors. The integrating factor is $\mu(x) = e^x$. The solutions are $y = -\frac{1}{2}e^x + Ce^{-x}$.
- (j) This equation is exact. The solutions are $xy + x \cos y - \sin x = C$.

7. Let $x(t)$ = amount of salt in lbs in the tank at time t minutes.

The concentration of the tank at any given time is $\frac{x(t)}{100}$ lb/gal.

$$\begin{aligned} \Rightarrow \quad \frac{dx}{dt} &= (0.2 \text{ lb/gal})(30 \text{ gal/min}) - (\frac{x}{100} \text{ lb/gal})(30 \text{ gal/min}) \\ &\Rightarrow \quad \frac{dx}{dt} = 6 - 0.3x \quad \text{in lb/min} \end{aligned}$$

We have the initial condition $x(0) = 0$.