Differential Equations - Exam #1 Practice Problems

These practice problems are a collection from other exams from other instructors.

1. What can you conclude about the existence and uniqueness of the solution(s) to the initial value problem $\frac{dy}{dx} = y^{1/3}, \quad y(0) = 2$?

2. What can you conclude about the existence and uniqueness of the solution(s) to the initial value problem $\frac{dy}{dx} = y^{1/3}, \quad y(0) = 0$?

3. Consider the differential equation $\frac{dy}{dx} = (y^2 - 3y + 2)(y - 3)$. Determine the equilibrium solutions for the differential equation. Roughly sketch the direction field and sketch several graphs of solutions in the xy-plane.

4. Consider the differential equation $\frac{dy}{dx} = xy$. Roughly sketch the direction field and sketch several graphs of solutions in the xy-plane.

5. Consider the initial value problem $\frac{dy}{dx} = 3x - 2\sqrt{y}, \quad y(0) = 1$. Find the approximate value of $y(0.3)$ by Euler's method with step size $h = 0.1$.

6. Solve the following equations. Some are initial value problems.

   (a) $\frac{dy}{dx} = \frac{(2y^2+1)(x^2+\cos x)}{4y}$ with $y(0) = \frac{\sqrt{2}}{2}$

   (b) $\frac{1}{r}\frac{dr}{d\theta} + \tan\theta = \frac{1}{r}\sec\theta$ with $r(0) = 2$

   (c) $(y^2 - 2x)dx + (2xy + 1)dy = 0$

   (d) $(2xy^3 + y^4)dx + (xy^3 - 2)dy = 0$

   (e) $\frac{dy}{dx} = \frac{2(x+2y)}{x-y}$

   (f) $\frac{dy}{dt} + \cos(t)y = e^{-\sin t} + \cos t$

   (g) $(x^2 - y^2)dx + xy\ dy = 0$

   (h) $\frac{dy}{dx} = \frac{\sin x + 1.5e^x}{y-1}$ with $y(0) = -1$

   (i) $(y + e^x)dx + dy = 0$

   (j) $x(1 - \sin y)\frac{dy}{dx} = \cos x - \cos y - y$

7. A 100 gallon tank initially contains pure water. A salt mixture with a concentration of 0.2 lb/gal is pumped into the tank at the rate of 30 gal/min. The well-stirred mixture leaves the tank at the same rate that it flows in. Set up, a differential equation to model this mixing problem and give an initial condition. Be sure to define any variables you use. You do not need to solve the equation.