Math 207
Exam 1 Answers

1. (a) $\dfrac{dy}{dx} = 4x^3 - 10 - \dfrac{4y}{x} \quad \Rightarrow \quad f(x,y) = 4x^3 - 10 - \dfrac{4y}{x}$

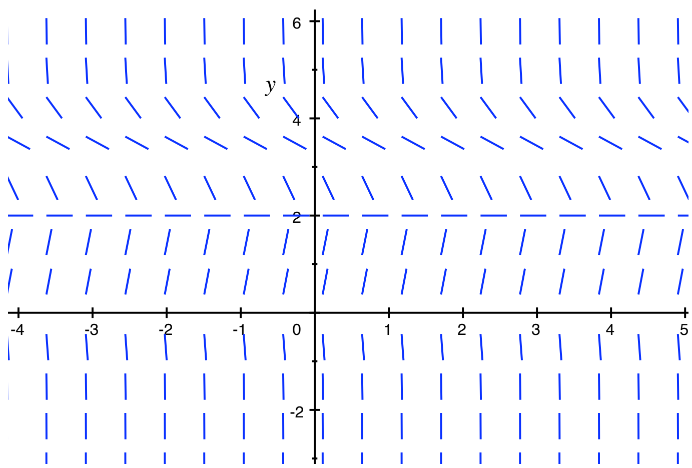
   Since $f(x,y)$ and $\dfrac{\partial f}{\partial y}$ is continuous at (1,5), $\boxed{\text{there is a unique solution}}$ to the IVP.

   (b) This is a linear Equation. To solve, use an integrating factor of $\mu(x) = x^4$.

   $\Rightarrow \quad \boxed{y = \frac{1}{2}x^4 - 2x + \frac{13}{2}x^{-4}}$

2. (a) $\boxed{y \equiv 0, \; y \equiv 2, \; y \equiv 4}$

   (b) The following graph isn't very good. (I can't get my graphing program to work well.) The main things needed are that we have slope 0 along the lines $y = 0, \; y = 2, \; y = 4$. We have positive slope when $0 < y < 2$ and negative slope everywhere else.



   (c) If the initial condition is $y(0) = 5$, then we are beginning at a point above the line $y = 4$. Looking at the direction field and by the existence and uniqueness of solutions, we can see that $\lim_{x \to \infty} y(x) = \boxed{4.}$

3. $\boxed{\text{Yes}}$ If you implicitly differentiate the original equation ($xy = \ln(y) + x$), you can show that it satisfies the differential equation.

4. (a) This equation is separable and it separates as follows: $\quad e^{-x}\, dx = t^2 e^{t^3}\, dt$

   Integrating both sides (using substitution): $\quad \boxed{-e^{-x} = \frac{1}{3}e^{t^3} + C}$

   (b) This equation isn't exact, but it can be made into an exact equation by using a special integrating factor. Since $\dfrac{\partial N/\partial x - \partial M/\partial y}{M} = -\dfrac{2}{y}$ depends only on y, we can use the special integrating factor of $\mu(y) = y^{-2}$.

$$\Rightarrow \quad (2\sin x \cos x + 2xy^{-1})dx \ - \ x^2y^{-2}dy \ = \ 0 \quad \text{is exact}$$

Solution: $\boxed{\sin^2 x + x^2y^{-2} = C, \ \text{ and } \ y \equiv 0}$

5. Diff. EQ: $\boxed{\dfrac{dB}{dt} = 0.04B - 30,000}$ Initial Condition: $\boxed{B(0) = 750,000}$