

**Math 207**  
**Exam 1**  
**April 22nd, 2011**

Name: \_\_\_\_\_

1. Your exam contains 6 questions and 6 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 50 minutes for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. Leave answers in exact form (as simplified as possible).
5. Put a box around your final answer where applicable.
6. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic). You are allowed one 4"×6" notecard (both sides).
7. If you need extra space, attach a sheet to the back of the exam and clearly indicate this.
8. Note: Do not be intimidated by the amount of space provided! I wanted to ensure that you had more than enough space for each problem. This does not mean that I expect you to fill the space.
9. **Define all variables you use** when writing differential equations or initial value problems.

Problem	Total Points	Score
1	21	
2	14	
3	9	
4	30	
5	15	
6	11	
Total	100	

1. (21 pts.) For parts (a) and (b) below, consider the initial value problem  $\frac{dy}{dx} + \frac{4y}{x} + 10 = 4x^3$ ,  $y(1) = 5$ .

(a) (6 pts.) Determine whether the existence-uniqueness theorem implies that the given initial value problem has a unique solution.

(b) (15 pts.) Solve the initial value problem. Express your solution(s) explicitly.

2. (a) (4 pts.) Write the equations of the equilibrium solutions of  $\frac{dy}{dx} = y(2-y)(y-4)^2$ .

(b) (6 pts.) Sketch a direction field of  $\frac{dy}{dx} = y(2-y)(y-4)^2$ .

(c) (4 pts.) For the initial value problem,  $\frac{dy}{dx} = y(2-y)(y-4)^2$ , with  $y(0) = 5$ , what is the  $\lim_{x \rightarrow \infty} y(x)$ ?

3. (9 pts.) Yes or No: Is the implicitly defined relation,  $xy = \ln(y) + x$ , a solution to the equation

$$(xy - 1)\frac{dy}{dx} = y - y^2? \quad (\text{Show work.})$$

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4. (30 pts.) Solve the equations in (a) & (b). You can leave **both answers written implicitly**.

(a) (15 pts.)  $\frac{dx}{dt} - t^2e^{x+t^3} = 0$  (Note: The exponent of  $e$  is  $x + t^3$ .)

#4 Continued on the Next Page →

#4 Continued:

(b) (15 pts.)  $(2y^2 \sin x \cos x + 2xy)dx - x^2 dy = 0$

5. (15 pts.) Show that the equation  $\frac{dy}{dx} = \frac{y^2+xy+x^2}{x^2}$  is homogeneous and then solve the equation. You may express your solution(s) implicitly.

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6. (11 pts.) A bank account grows continuously at an annual rate of 4% and the account is also decreased continuously by \$30,000 each year. The account initially has \$750,000.

If  $B$  is the balance of the account at year  $t$ , **set up, but do not solve**, an initial value problem (Diff. EQ and an initial condition) to model this situation.