Math 207 Quiz #9 Answers

1. (a) Spring Constant: $l = \dfrac{mg}{k} \quad \Rightarrow \quad k = \dfrac{mg}{l} = \dfrac{4(9.81)}{1.635} = 24$ N/m

   Differential Equation: $4y'' + 3y' + 24y = 6\cos(2t)$ This system is underdamped ($b^2 < 4mk$)

   The steady-state solution is the particular solution $y_p = \dfrac{6}{\sqrt{(24-4(2)^2)^2+(3)^2(2)^2}}\sin(2t+\phi)$

   $\boxed{\Rightarrow \quad y_p = \frac{3}{5}\sin(2t+\phi)}$

   where $\tan\phi = \dfrac{24-4(2)^2}{3(2)} = \dfrac{4}{3}$ with ϕ in quadrant I $\Rightarrow$ $\boxed{\phi = \arctan(\frac{4}{3}) \approx 0.9273.}$

   (b) Since $\gamma_r = \sqrt{\frac{24}{4} - \frac{3^2}{2(4)^2}} = \sqrt{\frac{183}{32}} = \frac{\sqrt{366}}{8}$, the resonance frequency is $\boxed{\dfrac{\gamma_r}{2\pi} = \dfrac{\sqrt{366}}{16\pi} \approx 0.3806 \text{ cycles/sec}}$

2. Here's just one way to solve this. I will use substitution, but you could also use elimination.

   Solving for y in equation 1: $y = x - x'$

   Substituting this into equation 2 yields a lovely homogeneous equation in x: $x'' - 2x' - 3x = 0$

   Solution of the homogeneous equation: $\boxed{x(t) = c_1 e^{3t} + c_2 e^{-t}}$

   Using the substitution equation $y = x - x'$ to find y: $\boxed{y = -2c_1 e^{3t} + 2c^2 e^{-t}}$